AL.3.2010-102
University of Alberta Library
0 1620 3452443 7

Mathematics 8

LearnEveryWare

Unit 5

Percent, Ratio, and Rate

we encourage

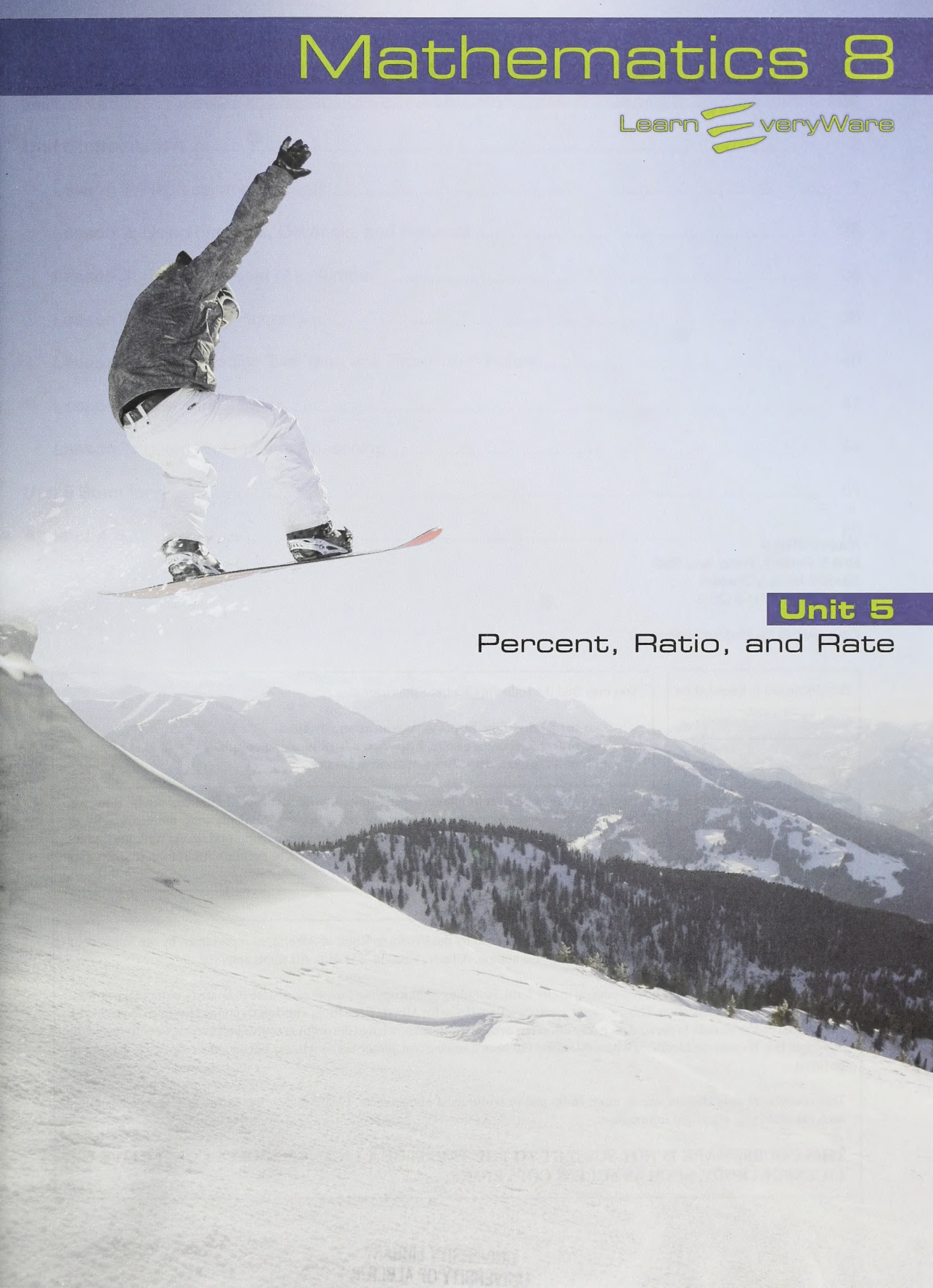
Alberta Education

Digitized by the Internet Archive
in 2016 with funding from
University of Alberta Libraries

https://archive.org/details/mathematics805albe

Mathematics 8

LearnEveryWare



Unit 5

Percent, Ratio, and Rate

Mathematics 8
Unit 5: Percent, Ratio, and Rate
Student Module Booklet
ISBN 978-0-7741-3135-3

Cover Art: © Ilja Mašík/shutterstock

This document is intended for	
Students	✓
Teachers	✓
Administrators	
Home Instructors	
General Public	
Other	

You may find the following Internet sites useful:

- Alberta Education, **http://www.education.gov.ab.ca**
- Learning Resources Centre, **http://www.lrc.education.gov.ab.ca**
- Tools4Teachers, **http://www.tools4teachers.ca**

Exploring the electronic information superhighway can be educational and entertaining. However, be aware that these computer networks are not censored. Students may unintentionally or purposely find articles on the Internet that may be offensive or inappropriate. As well, the sources of information are not always cited and the content may not be accurate. Therefore, students may wish to confirm facts with a second source.

Copyright © 2009, Alberta Education. This resource is owned by the Crown in Right of Alberta, as represented by the Minister of Education, Alberta Education, 10155 – 102 Street, Edmonton, Alberta, Canada T5J 4L5. All rights reserved.

No part of this courseware may be reproduced in any form, including photocopying (unless otherwise indicated), without the written permission of Alberta Education. This courseware was developed by or for Alberta Education. Third-party content has been identified by a © symbol and/or a credit to the source. Every effort has been made to acknowledge the original source and to comply with Canadian copyright law. If cases are identified where this effort has been unsuccessful, please notify Alberta Education so corrective action can be taken.

This courseware may contain one or more audio and/or multimedia components. Please review the Terms of Use Agreement on each for additional copyright information.

THIS COURSEWARE IS NOT SUBJECT TO THE TERMS OF A LICENCE FROM A COLLECTIVE OR LICENSING BODY, SUCH AS ACCESS COPYRIGHT.

UNIVERSITY LIBRARY
UNIVERSITY OF ALBERTA

Contents

Unit 5 Introduction 2

Lesson 1: Representing Percents 7

Lesson 2: Using Fractions, Decimals, and Percents 15

Lesson 3: Finding Percent of a Number 26

Lesson 4: Combining Percents 33

Lesson 5: Understanding Two-Term and Three-Term Ratios 40

Lesson 6: Comparing Rates 47

Lesson 7: Using Proportional Reasoning 54

Unit 5 Summary 61

Appendix 69

Unit 5: Percent, Ratio, and Rate

Unit 5 Introduction

© Jeremy Edwards/iStockphoto
Moraine Lake is located near the community of Lake Louise.

Alberta has some of the most scenic lakes in the world. In order to protect Alberta's natural heritage, scientists study the wildlife and water resources in the province. They often use percents, ratios, and rates to communicate their findings.

The findings tell us that about 10% of this province is covered with fresh water. Most of our lakes lie in the northern half of the province. Lake Athabasca, the largest lake in Alberta, has only 30% of its area in the province. Lake Clare is the largest lake wholly within Alberta's borders. Pakowki Lake is the largest lake in southern Alberta, but it is only 9.3% of the size of Lake Clare.

The marshy shorelines of Pakowki Lake, however, are a haven for migratory waterfowl and shorebirds. Northern pintail ducks have been recorded there at ratios of 5 : 1 to the longbilled dowitchers, a type of shorebird. Yet a 1996 survey showed that Pakowki Lake was home to 4% of the estimated global population of dowitchers.

The Bow River is one of the finest trout-fishing rivers in the world. The highest flow rate of the river is in June, averaging around 240 cubic metres per second. By the end of August, the flow usually drops to less than 25% of that flow rate.

In this unit you will be working with percentages, ratios, and rates involving water and wildlife. As well, you will use rates and proportional reasoning to compare situations and find answers to questions. You will develop understanding and skill in dealing with percentages less than 1% and greater than 100%. As you progress in the unit, you will solve real-life problems with proportional reasoning using ratios and rates. Just like scientists, you will communicate your information using percents, ratios, and rates.

This unit will help you answer the following critical question: How can you use percentages, ratios, and rates to make meaningful comparisons when investigating water conservation and natural resources?

The new concepts and skills with percent, ratio, rate, and proportion will be presented in seven lessons.

Mathematics 8 LearnEveryWare

Unit 5: Percent, Ratio, and Rate

Lesson 1: Representing Percents

Lesson 2: Using Fractions, Decimals, and Percents

Lesson 3: Finding Percent of a Number

Lesson 4: Combining Percents

Lesson 5: Understanding Two-Term and Three-Term Ratios

Lesson 6: Comparing Rates

Lesson 7: Using Proportional Reasoning

In this unit there are a variety of assignments you will be asked to complete. Some of these include:

- posting to the discussion board
- adding samples of your work to your Math 8 folder
- completing sets of questions for each lesson
- solving a unit problem at the end of the unit
- completing evaluation pieces assigned by your teacher

Strategies for Success

In order to support your success in this unit, follow these strategies.

Strategy 1

Make foldable study tools according to the detailed instructions on pages 44 and 120 of your textbook. Although your foldables may not be graded, you will benefit from these tools. Keep these points in mind as you develop and use these study tools:

- Add formulas, examples, and vocabulary words as you are work through the lessons.
- The foldables can serve as quick reference guides and will help you save time when you are ready to study for your unit test.

Strategy 2

In this unit you will be referring to pages 42 to 75 and 118 to 153 of your textbook.

- Take time to flip through these textbook pages.
- Look at illustrations, margin features, and main titles to get a sense of where you will be going.

Strategy 3

Read your lessons and textbook materials carefully.

- Pay special attention to graphs, tables, and diagrams. These items are provided to help you learn concepts.
- Read and reread material, and take time to understand it.
- Ask yourself these questions: What material is new to me? What do I already know?
- Move ahead with confidence. You do have the skills to understand the textbook's meanings.

Unit 5 Problem

You will conclude this unit with a problem. You are required to develop a water conservation plan and report the expected water savings for the plan. Your presentation will also include statistics on the rate of glacier melting and its predicted effect on future river flows.

Look at "Math Link" on page 121 and "Wrap It Up!" on page 153 of your textbook. In addition to the directions there, you will be asked to calculate the rate of glacial melting from statistics and to predict how that will influence river flows.

At the end of numbered sections in the textbook, you will see "Math Link" sections. Some "Math Link" sections feature questions involving water conservation. Answering these questions will help you complete the Unit 5 Problem.

Unit 5: Percent, Ratio, and Rate

Lesson 1: Representing Percents

Get Focused

© redfrisbee/shutterstock
An inukshuk is built along the bank of a river near Jasper.

Many people in Alberta have strong concerns about the need to conserve water resources. Among the people who are concerned about water conservation are Aboriginal people in the province. The inukshuk pictured above is a symbol of the Inuit people. According to the 2006 census, about 6% of Albertans are Aboriginal. Of those, only 0.9% of Aboriginal people in Alberta are Inuit.

Among the cities in Canada, Edmonton has the second largest percentage of Aboriginal citizens. The Inuit population in Edmonton in 2006 was 128% of that reported five years earlier. However, the Inuit in Edmonton are still only about $\frac{1}{20}\%$ of the national Aboriginal population. Statisticians think that the rapid rise in the number of Aboriginal people reported across Canada may be partly due to people having increased pride in their Aboriginal heritage. More Canadians with Aboriginal roots now indicate it on their census forms.

From your earlier years of math study, you are likely familiar with measuring values in percents. However, values of more than 100% and fractions of percents may seem new to you at this

point. In this lesson learning to represent percents on a grid will strengthen your understanding of percents. By the end of this lesson you will be able to represent very large and very small percents with ease.

This lesson will help you answer the following critical question: How can a percent value less than 1% or greater than 100% be visually represented?

You can complete the activities in this lesson using either a ruler and coloured pencils or a computer with a drawing program such as MS Word. You will need to use 10 by 10 grid paper, no matter which method you choose. (You may see the 10 by 10 grids described as *hundred grids*.)

Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 1 Question Set

Explore

Can you represent percents on a grid? Give it a try by working through "Explore the Math" on pages 122 and 123 of the textbook. Carefully read the "percent" note in the margin because it will help you correctly complete your work on the grids.

In the "Explore the Math" activity, you will discover the following:

- Question 1 deals with percents that are 100% and greater.
- Question 2 deals with percents that are between 1% and 100%.
- Question 3 deals with percents that are less than 1%.
- Question 4 allows you to summarize what you have discovered.

Try This

TT 1. Complete "Explore the Math" questions 1, 2, 3, and 4 on pages 122 and 123. Working with a partner in this activity will help you learn these math concepts. Ask your teacher about who an appropriate partner would be. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

If you want to do the shading using your computer, go to the Math 8 Multimedia DVD, and download the "10 by 10 grid paper."

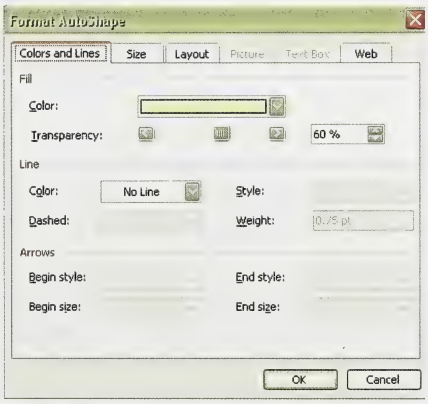
Directions for working with AutoShapes are found in "MS Word" available on the Math 8 Multimedia DVD. Tips are provided for both MS Word 2007 and MS Word 2003. Choose the one that matches your computer. Drawing programs are included with most Mac computer systems, and the "Help" section will show you how to use the rectangle shape.

In addition to those directions, read Rectangle Shapes below for more tips to complete this question.

Rectangle Shapes

You may find the following steps helpful:

- Use one or more of the rectangle shapes to shade over the required squares on a grid.
- Double-click on a rectangle after you have drawn it and choose the "Colors and Lines" tab.



Microsoft product screen shot(s) reprinted with permission from Microsoft Corporation.

- Under "Fill," select the colour you want. Set the transparency of the colour at about 60% so you will be able to see the grid lines.
- You can set the colour and transparency of several rectangles at once, if you hold down the shift key and click on all the rectangles in turn. Then, in the toolbar at the top of your screen, select "Format," followed by "AutoShapes." Go to the "Colors and Lines" tab, as above.
- If you want to remove the edge lines, in the Line section choose "No Line."

To demonstrate your understanding, complete the following Try This questions.

TT 2. Show how to represent 128% of a population, mentioned in the Get Focused part of this lesson, using 10 by 10 grids. Print out another page of the grids, if needed. Remember, 10 x 10 grids are available on the Math 8 Multimedia DVD.

TT 3. Show how to represent 0.9% of a population, mentioned in the Get Focused part of this lesson, using 10 by 10 grids.

TT 4. The Inuit represent 4.3% of Aboriginal people in Canada. Show how to represent 4.3% of a population using 10 by 10 grids.

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Create your own percent to be represented on the 10 by 10 grid paper, and post this to the discussion board, along with your responses to TT 2, TT 3, TT 4, and TT 5. Then respond to at least two other student postings by completing grids for the percents they created.

Read

How sure were you that the fractions of squares in the Try This activity were the right size? To increase your confidence, find a more sure method of representing fractional squares by reading "Example 1: Determine the Percent Represented on a Grid" on pages 124 and 125 of the textbook. See how to put a grid in the expanded square to clearly show the fraction.

Self-Check

SC 1. When you have finished the reading, answer "Show You Know" on page 125 of the textbook.

Compare your answers in the Appendix.

Read

You have seen how to read percents from a grid, even fractional and decimal percents. Get some more pointers by reading "Example 2: Represent Percents on a Grid" on pages 126 and 127 of the textbook.

Self-Check

SC 2. Answer "Show You Know" on page 127 of the textbook.

Compare your answers in the Appendix.

Read

Carefully read "fractional percent" in the margin on page 126 and "Key Ideas" on page 127 of the textbook. Transfer the important ideas to your foldable. The directions to build a foldable are found on page 120 of the textbook. Step 3 of the directions means to cut out three 10 by 10 grids and not to cut out 300 grids, as the textbook seems to be saying.

Connect

Now that your skills with representing percents in a graphical format are stronger, how about applying them to the world around you? Put your new-found mathematical strength to work by completing the Self-Check questions below.

Self-Check

SC 3. Complete "Check Your Understanding" questions 4, 6, 8, 10, and 11 on pages 128 and 129 of the textbook. If you are having difficulty on any answers, discuss them with your partner or your teacher until you are sure you know how to do that type of question accurately. It's smart to ask questions.

Compare your answers in the Appendix.

Extra Practice

If you were able to accurately answer at least five of the previous questions, and you feel you have a solid understanding of how to read percents off a grid and how to represent fractional percents on a grid, go on to the assignment. If you need a bit more practice with the concepts and ideas of this lesson, you may complete questions 5, 7, 9, and 12 from "Check Your Understanding" on pages 128 and 129 in the textbook. When you finish each question, check your work using the shortened answers on page 486 at the back of the textbook. Make sure to contact your teacher if you still have questions after completing this additional practice.

Assignment

To inform your teacher of your understanding of the ideas in this lesson, this question set will need to be completed and submitted to your teacher. In order to complete question 2, you may find it helpful to go to the Math 8 Multimedia DVD and print three copies of "Three Grids." You can attach your completed copy to the Question Set. For questions 3, 4, and 5, you may find it helpful to go to the Math 8 Multimedia DVD and print as many copies as you need on "One Grid."

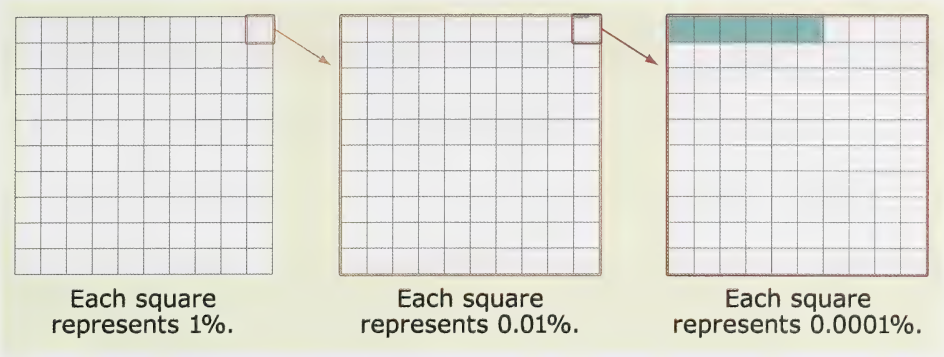
Go to the Unit 5 Assignment Booklet, and complete "Unit 5: Lesson 1 Question Set."

Going Beyond

Sometimes percents are extremely small. "Math Link" on page 129 of the textbook shows some fresh water percents that are less than 0.1%. How would you show those so that someone less knowledgeable than you could easily understand them?

"Extend" question 15 on page 129 of the textbook asks you to represent a very small percent—0.000 0125%. The answer key on page 486 for that question suggests placing many 10 by 10 grids (called *hundred grids*) together. That would be a very large page if you attached 100 000 hundred grids to create 10 million squares.

Another way to represent a very small percent would be to expand several expansion squares in a row. For example, in order to represent 0.0006%, enlarge one square into a 10 by 10 grid; then enlarge one of those squares into another 10 by 10 grid, as shown on the next page:



The value of 0.0006% is coloured in on two of the grids.

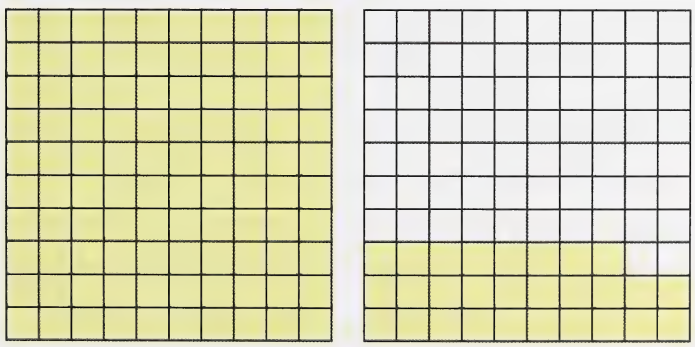
Go ahead and represent the fresh water percents less than 1% given in "Math Link" on page 129 of the textbook using grids as needed.

Compare your answers in the Appendix.

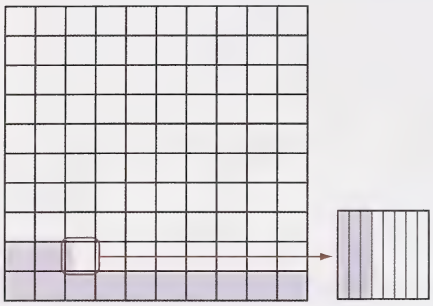
Lesson Summary

Percents can be used to compare many things, including population statistics and the size of fresh water lakes. In this lesson you represented percents greater than 100% and fractional percents with grids of a hundred squares each.

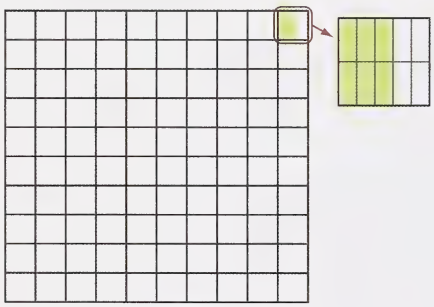
Percents greater than 100% are shown using more than one grid of a hundred squares. To show that the 2006 Edmonton Inuit population is 128% of the 2001 census report, use two grids and shade them as shown below:



Fractional percents can be represented by shading in part of a grid square. Zooming in on the square that represents a fractional percent can give an enlarged and detailed view of the fraction involved. For example, $12\frac{3}{8}\%$ can be represented by the following diagram.



Fractional percents less than 1% are represented by shading in part of one square of a hundred-square grid. Show an enlarged view of the shading in that one square for clarification. For decimal fractions of a percent, the enlarged square would be divided into tenths or hundredths, with the corresponding portion shaded. For example, 0.6% is $\frac{6}{10}\%$ and would be shown as follows:



Unit 5: Percent, Ratio, and Rate

Lesson 2: Using Fractions, Decimals, and Percents

Get Focused

© Don Wilkie/iStockphoto

Brazeau Dam Power Plant is built on the Brazeau River. This hydroelectric power plant is located about 55 kilometres southwest of Drayton Valley.

With water rushing down from the mountains on Alberta's western border, it would be easy to think that most of the electricity is generated from this source. Instead, only 7.1% of Alberta's electricity came from hydroelectricity in 2008. Coal-fired power plants produced nearly $\frac{1}{2}$ of the electricity, and gas-fired plants about 0.38 of the province's power. Wind generators are becoming more popular, yet wind generators only made $\frac{1}{24}$ of the electricity. The remaining 1.5% of electricity production came from biomass.

© Andrew Penner/iStockphoto

In the preceding paragraph, values were presented as percents, as proper fractions, and as decimals. Media and advertising give values in a variety of forms. In our modern world, it helps if you know how to convert from one to another. Then you can compare and make sense of what you read and hear. By the end of this lesson, expect your skill at converting back and forth between fractions, decimals, and percents to dramatically increase.

This lesson will help you answer the following critical question: When given a value in the form of a fraction, decimal, or percent, how can you express the value in another form?

You may need a calculator for this lesson.

Assignments

Your assignment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 2 Question Set

Explore

How strong is your present skill at converting between fractions, decimals, and percents? Give it a try by doing "Explore the Math" on pages 130 and 131 of the textbook.

Working with a partner in this activity will help you learn these math concepts. Ask your teacher about who an appropriate partner would be. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

Try This

TT 1. Answer "Explore the Math" questions 1, 2, 3, and 4 on pages 130 and 131 of the textbook.

My Guide

Q: In question 1.b), how do you find a denominator of 100?

A: Think about how to get a denominator of 100. For example: $100 \div 20 = 5$. Suppose each of the small rectangles in the square were divided into five smaller rectangles. Then there would be 45 shaded ($9 \times 5 = 45$) and 100 unshaded ($20 \times 5 = 100$) smaller rectangles.

To demonstrate your understanding, complete the following Try This questions. Work with a partner on these if you can.

TT 2. At the beginning of this lesson, Alberta's power produced by natural gas was given as the decimal fraction 0.38.

a. Express 0.38 as a fraction with a denominator of 100.

b. Give the fraction also in lowest possible terms, if it can be reduced.

c. Express 0.38 as a percent.

TT 3. Coal is used to produce nearly ½ of our power.

a. Express the fraction ½ as a fraction with a denominator of 100.

b. Express the fraction ½ as a decimal.

c. Express the fraction ½ as a percentage.

TT 4. The hydroelectric portion of Alberta's power was given as 7.1%.

a. Express 7.1% as a decimal.

b. Express 7.1% as a fraction with a denominator of 1000.

After you have discussed your work with a partner, if that was possible, place a copy of your answers in your Math 8 course folder. If you had no partner, look the answers over carefully, and then put them into your folder.

Watch and Listen

If you feel a need to brush up on working with decimals, go to the Math 8 Multimedia DVD, and open "Decimals" for an interactive learning experience.

If understanding percent is still a challenge, go to the Math 8 Multimedia DVD and open "Percent" to strengthen your skills there.

Read

Was it easy to convert from one form to another? To increase your skills and your confidence, read "Example 1: Convert Fractions to Decimals and Percents" on page 131 of the textbook. Pay special attention to how the numerator (the top number) and denominator (the bottom number) are placed when a fraction is converted to a decimal. For example, in the fraction $\frac{5}{8} = 5 \div 8 = 0.625$, where were the numerator and denominator positioned to perform the calculation? Can you see how the numerator (top number) is divided by the denominator (bottom number)?

Note how a decimal is changed to a percent by multiplying by 100 (e.g., move the decimal two places to the right) and adding the percent symbol. For example: 0.625 = 62.5%.

Self-Check

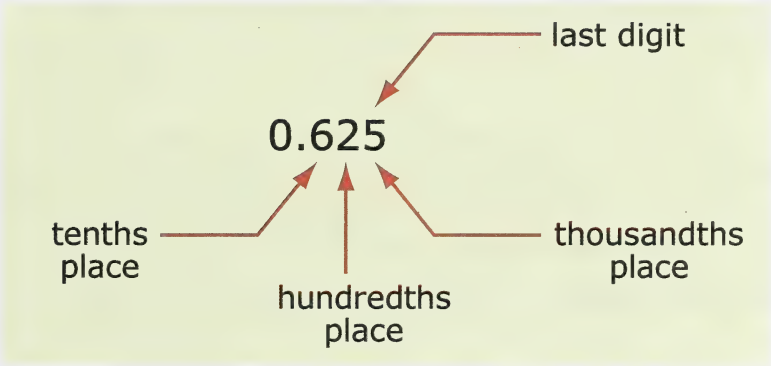
SC 1. When you have finished reading "Example 1," answer the "Show You Know" on page 132 of the textbook to make sure you are able to accurately make the conversions.

Compare your answers in the Appendix.

Read

You have seen how to convert fractions to decimals and to percents. How are your skills at converting decimals to percents and to fractions? Get some more pointers by reading "Example 2: Convert Decimals to Percents and Fractions" on page 132 of the textbook. See how the decimal is converted to a percentage by multiplying by 100.

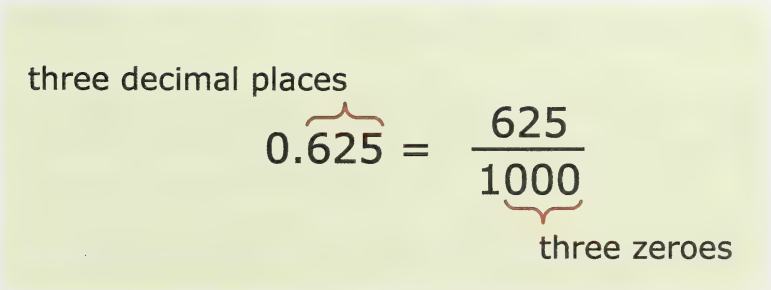
To convert a decimal to a fraction, you must look at the last digit in the decimal and determine its place value (e.g., tenths, hundredths, thousandths). Then multiply the numerals in the decimal by whatever the place value is. For example, in 0.625, the last digit, 5, is in the thousandths place.



So multiply 625 by one one-thousandth ($\frac{1}{1000}$) to get a fraction.

$0.625 = 625 \times \frac{1}{1000} = \frac{625}{1000}$

It may simplify the strategy to think of the process as "there are three decimal places, so divide the three numerals by the multiple of 10 with three zeros."



Note also how the fraction is then reduced to its lowest terms by dividing both the numerator and denominator by the largest number that will divide evenly into both.

$$\frac{625}{1000} = \frac{625 \div 125}{1000 \div 125} = \frac{5}{8}$$

Sometimes you will have to divide by several smaller numbers in a row to get the fraction into its lowest terms because you don't know the largest number that will divide into both the numerator and denominator. In the above example, you may have divided both the numerator and denominator by 25 and then by 5 to get it to its lowest terms.

$$\frac{625}{1000} = \frac{625 \div 25}{1000 \div 25} = \frac{125 \div 5}{400 \div 5} = \frac{5}{8}$$

Keep going until no more numbers will divide evenly into the numerator and denominator. Doing it in several division steps is perfectly OK.

Self-Check

SC 2. Answer the "Show You Know" on the bottom of page 132 of the textbook.

Compare your answers in the Appendix.

Read

The third skill is to convert percents to fractions and to decimals. Read "Example 3: Convert Percents to Fractions and Decimals" on page 133 of the textbook. See how a percent is converted to a fraction by writing the percent as the numerator over a denominator of 100. Then the fraction must be reduced to its lowest terms. An example follows.

$40\% = \frac{40}{100} = \frac{2}{5}$ (Divide both numerator and denominator by 20.)

Be sure to carefully read the notes in the margin. They give pointers about reducing fractions and getting rid of decimal places in fractions. Put the strategies that are new into your foldable. That way, the new ideas you will need to review are there at your fingertips.

You will see that the easiest conversion is to express a percent as a decimal. Simply divide the percent by 100 (e.g., move the decimal two places to the left) and drop the percent symbol.

Self-Check

SC 3. Answer the "Show You Know" on page 133 of the textbook. If you have any difficulty with these conversions, be sure to discuss it with your teacher. Sometimes one simple clue is all you need to be on track.

Compare your answers in the Appendix.

Read

At last, you come to the final skill—finding percents from numbers. Now you are applying the previous skills to real life. Read "Example 4: Determine a Percent" on page 134 of the textbook.

When you are asked to find the percent of something, the number representing that something will be the denominator in a fraction (e.g., the number you divide by). It represents 100% in the fraction. After you divide the fraction, convert the decimal answer to a percent by multiplying by 100 and adding the percent sign.

For example, the sale price of a sofa is $200 less than the original price of $800. What percent is the discount of the original price?

You are asked to find the percent of the original price, so you will divide by the original price.

$$\frac{200}{800} = 200 \div 800 = 0.25 = 25\%$$

The discount is 25% of the original price.

Self-Check

SC 4. Answer the "Show You Know" on page 134 of the textbook. If something is not making sense in an answer, please ask your teacher about it.

Compare your answers in the Appendix.

Discuss and Share

So far in this lesson, you have completed four Try This questions and four Self-Check questions. Which of these conversion questions was the trickiest for you? Explain how you worked through this problem, and post the explanation to the discussion board. Then look over one other student's post and let them know if you agree with their method and why.

Read

Carefully read "Key Ideas" on page 135. In your foldable, jot down the important ideas you have gained so far in this lesson.

Connect

Now that your skills with fractions, decimals, and percents are stronger, how about applying them to the world around you? Put your new-found mathematical strength to work by completing the Self-Check questions below.

Self-Check

SC 5. Do "Communicate the Ideas" questions 1 and 2 on page 135; then complete "Check Your Understanding" questions 4, 6, 8, 11, 13, 15, 16, and 18 on pages 135 through 137 of the textbook. Refer back to the examples as you do them. Check your work after each question using the answers given on page 487 at the back of the textbook. Before you complete question 16, first carefully study "Literacy Link" below the question. If you don't see how they arrived at any answer, please discuss it with your teacher.

Compare your answers in the Appendix.

SC 6.a. Complete "Math Link" on page 137 of the textbook.

b. As well as doing the math, think about how global warming may affect those percents. Answer the question, "If the frozen water melts into the ocean, how might the percents change?"

Compare your answers in the Appendix.

Extra Practice

If you feel you have a solid understanding of how to convert between fractions, decimals, and percents, go on to the assignment. If you feel you need a bit more practice, you may complete questions 3, 5, 7, 9, 10, 12, 14, 17, 19, and 20 from "Check Your Understanding" on pages 135 to 137 in the textbook. When you finish each question, check your work using the shortened answers given on page 487 at the back of the textbook. If you are still unclear about how to answer some questions, make sure you contact your teacher to ask about those questions and get some assistance.

Assignment

Go to the Unit 5 Assignment Booklet, and complete "Unit 5: Lesson 2 Question Set."

Going Beyond

Do you like puzzles that aren't too hard? Find the easy pattern in the table for "Extend" question 21 on page 137 by completing the first three rows. Do you see the pattern?

Go ahead and finish the last two rows. Check your work on page 487.

If you understand the pattern, complete three additional rows.

Compare your answers in the Appendix.

Now make up a puzzle similar to this one, and see if your classmates or family members can figure it out. Why not post your puzzle to the discussion board to challenge other students? Make sure you have the puzzle solved so that someone trying your puzzle can check his or her solution. Figure out other puzzles also, if there are some posted.

Lesson Summary

Percents are only one way of expressing a comparison or ratio. In this lesson you also used fractions and decimals. You converted back and forth between all three ways. There are six possible conversions:

- To convert a fraction to a decimal, divide the denominator (the bottom number) into the numerator (the top number).

  $\frac{5}{8} = 5 \div 8 = 0.625$

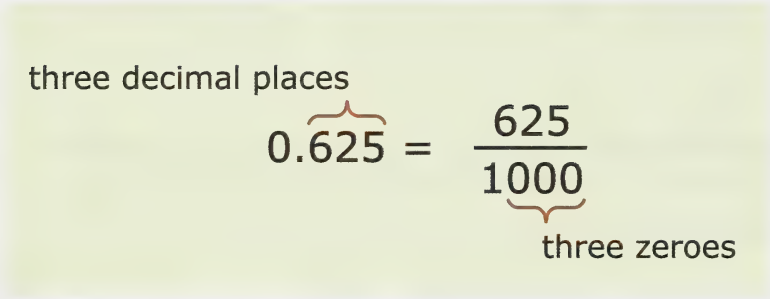
- To convert a decimal to a percent, multiply the decimal by 100 (e.g., move the decimal two places to the right) and add the percent sign.

  $0.625 = 62.5\%$

- To convert a decimal to a fraction, look at the last digit in the decimal and determine its place value (e.g., tenths, hundredths, thousandths). Then multiply the numerals in the decimal by whatever the place value is. For example in 0.625, the last digit, 5, is in the thousandths place. So multiply 625 by $\frac{1}{1000}$ to get a fraction.

  $0.625 = 625 \times \frac{1}{1000} = \frac{625}{1000}$

  It may simplify the process to think "there are three decimal places, so divide the numerals by the multiple of 10 with three zeros."



  Then express the fraction in its lowest terms by dividing both the numerator and denominator by the largest number that will divide into both evenly.

  $\frac{625}{1000} = \frac{625 \div 125}{1000 \div 125} = \frac{5}{8}$

  Sometimes, you will have to divide by several smaller numbers in a row to get the fraction into its lowest terms because you don't know the largest number that will divide into both the numerator and denominator. In the above example, you may have divided both the numerator and denominator by 25 and then by 5 to get it to its lowest terms. Doing it in several division steps is perfectly OK.

- To convert a percent to a fraction, write the percent as the numerator over a denominator of 100. Then reduce the fraction to its lowest terms.

  $40\% = \frac{40}{100} = \frac{2}{5}$ (Divide both numerator and denominator by 20.)

- To convert a percent to a decimal, divide the percent by 100 (e.g., move the decimal two places to the left) and drop the percent symbol.

- To convert a fraction to a percent, first convert the fraction to a decimal, as shown in the first bullet above. Then convert the decimal to a percent, as shown in the second bullet.

- When you are asked to find the percent of something, the number representing that something will be the denominator in a fraction (the number you divide by). It represents 100% in the fraction. After you divide the fraction, convert the decimal answer to a percent by multiplying by 100 and adding the percent sign.

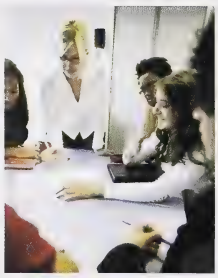
Unit 5: Percent, Ratio, and Rate

Lesson 3: Finding Percent of a Number

Get Focused

The Teen Environmental Action Club in a community is conducting a fundraiser. A significant percent of the garbage in their community consists of plastic grocery bags and cardboard. To cut down on plastic grocery bag waste, the club plans to sell 15 000 fabric grocery bags made from recycled plastic at $3.00 for two bags. With the profit from their sales, the club will purchase a cardboard baler to also reduce cardboard waste. The club's name and logo will be printed on the bags, so everyone in the community can see who has helped the club by buying bags. The club expects to make a profit of $34\frac{1}{2}\%$ on each bag.

How much money do they plan to make?



© Chris Schmidt/iStockphoto

To answer the question, you will have to make a calculation involving a fractional percent. The first lesson in Unit 5 enabled you to represent fractional percents on grids made with 100 squares. In Lesson 2 you learned how to convert back and forth between percents, decimals, and fractions. Now you will use those skills to solve problems involving percents. By the end of this lesson, expect your skill at solving problems about percents to be noticeably stronger.

This lesson will help you answer the following critical questions: How can you use both mental mathematics and a calculator to find the percent of a number?

You will need a calculator for this lesson.

Assignments

Your assessment will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 3 Question Set

Explore

Often you see sales discount banners and price tags indicating a percent discount. Usually you don't have a calculator handy to know the actual savings that are being offered, so it helps if you can mentally figure out the discount. How about learning to mentally calculate percentages?

Try This

TT 1. Complete "Explore the Math" questions 1, 2, 3, 4, and 5 on pages 138 and 139 of the textbook. You may use a calculator to verify your work after you have mentally made the calculation and recorded it.

Working with a partner in this activity will help you learn these math concepts. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

To demonstrate your understanding, complete the following Try This questions. Work with a partner on these, if you can, by discussing how to mentally figure out the answers in your heads.

TT 2. At the beginning of this lesson, the Teen Environmental Action Club planned to make a profit of $34\frac{1}{2}\%$ on each sale of \$3.00 for two bags. How can you mentally figure out 30% of \$3.00? What is the answer?

TT 3. How can you mentally figure out 4% of \$3.00? What is the answer?

TT 4. How can you mentally figure out $\frac{1}{2}\%$ of \$3.00? What is the answer?

TT 5. How can you mentally figure out $34\frac{1}{2}\%$ of \$3.00? What is the answer?

Place a copy of your answers in your Math 8 course folder.

Read

How about using mental math on a few other problems? Give "Example 1: Use Mental Math to Find the Percent of a Number" on pages 139 and 140 of the textbook a try. See how the textbook uses a series of easy steps to arrive at the answer, instead of trying to do it all at once. Be sure to carefully read "Literacy Link" in the margin on page 139 and the "Mental Math and Estimation (ME)" alternative strategy on page 140. They are guideposts to make the concepts easier to understand.

Self-Check

SC 1. When you have finished reading "Example 1," answer the "Show You Know" question on page 140 of the textbook to practise what you have read. There may be several good ways to arrive at the answer mentally. If you are unsure of your method of mentally finding the answer to a question, discuss it first with your partner and then ask your teacher about it.

Compare your answers in the Appendix.

Read

Sometimes the numbers may seem too complicated to do in your head. In that case, you can use pencil and paper or a calculator. Get some more pointers by reading "Example 2: Calculate the Percent of a Number" on pages 140 and 141 of the textbook. See how each part can be done in your head by reading "Literacy Link" on page 140 and the "Mental Math and Estimation (ME)" alternative strategies in the margins. Then you can decide whether to do the calculations mentally or with a calculator. Either way is correct.

You will see from the paper, pencil, and calculator solution that a fraction in the percent is first converted to a decimal by dividing the numerator (top number) by the denominator (bottom number). Then the whole percent is converted to a decimal by dividing by 100 (e.g., move the decimal two places to the left). Finally, the decimal is multiplied by the value following the word *of* to get the answer. (**Note:** The word *of* means "multiply" in math.)

For example, find $38\frac{3}{5}\%$ of \$2000.

- **Step 1:** A fraction is converted to a decimal.

  $3 \div 5 = 0.6$, so $38\frac{3}{5}\% = 38.6\%$

- **Step 2:** The total percent is converted to a decimal. (The percent was changed to a decimal by dividing by 100, which is easily done by moving the decimal two places to the left.)

  38.6% of \$2000 = 0.386 x \$2000

- **Step 3:** Multiply that decimal by the number in the question.

  0.386 x \$2000 = \$772

If you can do these percent questions in your head, similar to the solutions in the margins, the exercise will improve your brainpower. If calculator and pencil are the only way for you to go, that is all right too. These numbers are a little harder. The calculator steps are shown in bold print in the text for each question in "Example 2: Calculate the Percent of a Number."

Note that "Literacy Link" in the margin of page 140 tells you that *of* means to multiply. Now would be an excellent time to put those tidbits of knowledge into your foldable.

The "Mental Math and Estimation (ME)" boxes in the margins of the textbook pages show how these questions can be done in your head. Remember that working through these problems in your head, if possible, increases your "mental muscles," which will bring you big rewards when doing complex math in the future. Doing the problems in different steps than the textbook demonstrates is perfectly OK, as long as you arrive at the same answer as the textbook.

Self-Check

SC 2. When you have finished reading "Example 2," answer the "Show You Know" question on page 141 of the textbook to demonstrate to yourself that you have it down. If you are having difficulties with a question, discuss your challenges with a partner, if you can, and then with the teacher.

Compare your answers in the Appendix.

Read

Carefully read "Key Ideas" on page 141. Put the rest of the important ideas from this lesson into your foldable.

Connect

Now that your skills in using mental math to compute the percent of a number are stronger, how about applying them to the world around you? Put your new-found mathematical strength to work by completing the Self-Check questions below. You can use these Self-Check questions as a challenge to hone your mental math skills by completing the questions first in your head and then using the calculator to confirm your calculations.

Self-Check

SC 3. Complete "Check Your Understanding" questions 1, 3, 5, 7, 9, 11, and 13 on pages 142 and 143 of the textbook. Refer back to the examples if you need to as you do them. If you can't figure out a question or need help with some part, be sure to ask your teacher for help.

Compare your answers in the Appendix.

Try This

In preparation for your Unit Problem, you will need to research some ways to conserve water and discuss those strategies using percent. The "Math Link" at the bottom of page 143 will give you a good start. Use "Web Link" to begin your research.

TT 6. Find at least three ways to conserve water—one strategy each for in your home, in a school, and in your community. Then develop one water math problem for each of the three places. For example, if I replaced a 13 L/flush toilet with one that used only 6.0 L/flush, what percent of the water that goes down the toilet would I be saving? (Answer: 54%)

Extra Practice

If you were able to answer at least five of the SC 3 questions accurately without having to look at your notes or change your calculations, you may go on to the following Try This question. If, however, you need a bit more practice, complete questions 2, 4, 6, 8, 10, 12, and 14 from "Check Your Understanding" on pages 142 and 143 in the textbook to help you solidify the calculation process in your mind. When you finish each question, check your work using the answers given on page 487 at the back of the textbook. Be sure to ask your teacher for help if a question is giving you difficulty.

Try This

TT 7. Don't you find that some questions are like a puzzle, where you have to get the pieces of information to fit just right? Take a look at question 16 on page 143. In the second game, Josephine must have taken 10 shots, but not necessarily made a basket on each shot, or the question at the end is obvious. How many shots total did she take in the two games? Go ahead and figure out the number of baskets she made in the second game. Then check your work on page 487.

Discuss and Share

Were you able to work out the answer to question 16 in TT 7? If so, post an explanation to the discussion board describing how you worked through this problem. Then look over another student's post. Let the student know the similarities and differences you see in the two explanations.

If you were not able to solve question 16, choose two posts from students that did solve the question. Let the writers know how their explanations helped you better understand how to find a solution to the problem.

Assignment

To let your teacher know how well you understand the ideas set out in this lesson, complete this question set and submit it to your teacher.

Go to the Unit 5 Assignment Booklet, and complete "Unit 5: Lesson 3 Question Set."

Lesson Summary

In this lesson you learned to use mental math to find percents of numbers when the problem was not too complicated. This could be done by solving the problem in several smaller, more manageable steps. One of those smaller steps may be to figure out 10% of a number or 1% of a number. For instance, if a table saw is offered for sale at 45% off the regular price of $300, what are the savings? When you mentally figure out that 10% of $300 is $30, then 40% is easily computed (4 x $30 = $120). After that, 5% is half of $30 or $15. So 45% of $300 is $120 + $15, which is $135.

If the numbers are so complicated that a calculator must be used, it helps to remember that in mathematics the word *of* often means "multiply." Suppose you are asked to find $37\frac{5}{8}$% of 186 000.

The first step is to convert the fraction to a decimal using the skills from the previous lesson.

$\frac{5}{8} = 5 \div 8 = 0.625$ so $37\frac{5}{8}$% is 37.625%

The next step is to convert the complete percent to a decimal by dividing by 100, which is done by moving the decimal two places to the left, a strategy that was also learned in the previous lesson.

37.625% = 0.37625

Now comes the multiply step:

The phrase $37\frac{5}{8}$% of 186 000 means 0.37625 x 186 000. Using a calculator, it is 0.37625 x 186 000 = 69 982.5.

Therefore, $37\frac{5}{8}$% of 186 000 is 69 982.5.

The concepts in this lesson are built on an understanding of fractions, decimals, and percents gained in the last two lessons as well as your work in Unit 3.

Unit 5: Percent, Ratio, and Rate

Lesson 4: Combining Percents

Get Focused

© Ann Taylor-Hughes/iStockphoto

As you look at the renovations in the photograph of the bathroom, you may notice first the warm colours of the hardwood floor or the clean lines of the fixtures and tile. The most important water-saving feature, however, is the small size of the toilet tank. You probably don't usually think about it, but a typical family of four flushes 78 000 L of treated water down the drain in a year. That puts a strain on water treatment facilities and contributes to the concern over water shortages in Alberta, which you will be working on in your Unit 5 Problem. Installing an ultra-low flush toilet, such as the one pictured here, can reduce a family's toilet water consumption by 54%. If a family installs a dual-flush system instead, the family could reduce water consumption by 26%. How much water would the family save in a year if a dual-flush system was installed?

In the last lesson you learned how to find the percent of a number using just your brain or using a calculator. In this lesson you will learn to solve problems that involve taking the percent of a number more than once. In the example above, you could take 54% of 78 000 L, take 26% of 78 000 L, and then add the two products together. You will learn a couple of other strategies to deal with combined percents as you progress through this lesson.

Sometimes you must take a percent of a percent, instead of taking multiple percents of the original number. For instance, imagine finding a sale of 25% off on an item. Showing a certain card or coupon may save an additional 10% off the sale price. That will require a different strategy to figure out how much money is saved from the original price. By the end of this lesson, you will be able to decide on the appropriate strategy for the situation and to efficiently calculate the answers to problems involving combined percents.

This lesson will help you answer the following critical question: How can you calculate the combined effect of more than one change when each is expressed in percent?

You can complete the activities in this lesson by hand using a calculator, a ruler, coloured pencils, and 10 by 10 grid paper. Remember that "10 x 10 grid paper" is available on the Math 8 Multimedia DVD.You may also do your calculations using a computer. There is also a calculator on your computer and a drawing program, such as AutoShapes, in MS Word. If necessary, you can review the directions for working with AutoShapes by going to the Math 8 Multimedia DVD and opening "MS Word." Read about MS Word 2007 or MS Word 2003—the one that matches your computer.

Assignments

Your assignments will consist of the following:

- posting to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 4 Question Set

Explore

You have probably experienced buying something advertized at a percent discount and saw the clerk add on GST. Have you ever purchased a sale item in another province and had them add PST on top of GST? Could you figure out how much it would cost before you went to the till?

Try This

TT 1. Give that situation a try by working through "Explore the Math" questions 1, 2, 3, 4, and 5 on pages 144 and 145 of the textbook. Carefully read "Literacy Link" in the margin of page 144 so you are familiar with the meanings of GST and PST. Alberta and the territories do not have PST, but you will encounter provincial sales tax in most other provinces.

Working with a partner in this activity will help you learn these math concepts. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

You will need grid paper to answer two of the questions. If you do not have some, print 10 by 10 grid paper or download it if you are submitting your work online.

To demonstrate your understanding, complete the following Try This questions. If possible, discuss the questions with a partner as you work through them.

TT 2. At the start of this lesson, it was stated that a typical family of four runs 78 000 L of water through a regular toilet in a year. Assume that a family of three would use $\frac{3}{4}$ or 75% of that amount. What multiplication expression would you use to calculate the amount of water a typical family of three would use in a regular toilet?

TT 3. A study showed that installing an ultra-low flush toilet would give a typical family a 54% savings in flushed water. What multiplication expression could give you the yearly amount of water a typical family of three would save by using an ultra-low flush toilet?

TT 4. If the family of three chooses to install a dual-flush toilet instead, an additional 26% savings in flushed water can typically be expected. What percent of the water used by a normal toilet should they predict will be used to operate their toilet?

TT 5. What yearly amount of water should the family of three predict will be used to operate their dual-flush toilet?

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Post your responses to the TT questions onto the discussion board. Then find at least two other student postings that used a slightly different method than you to arrive at the answer. Respond to those postings by complimenting them on some aspect of their answers and asking them what led them to approach the problem the way that they did. It may be possible to learn something from their work that will benefit you in this unit.

Read

There are different ways to accurately answer problems where you combine percents such as GST and PST. See which method will work best for you as you carefully read "Example 1: Combined Percents" on pages 145 and 146 of the textbook.

The "Mental Math and Estimation (ME)" note on page 146 shows one way to answer the question in your head. Don't miss "Literacy Link" on page 146 that gives you a hint for methods 2 and 3.

The first method is to calculate each tax amount separately and then add them to the price of the item.

The second method is to add the percents together before figuring out the total tax and adding that to the price.

The third method is to add the percents for the taxes to the 100% of the price and then use that percent greater than 100% to directly calculate the total cost of the item.

Self-Check

SC 1. When you have finished reading "Example 1," answer the "Show You Know" question on page 146 of the textbook. Use the method you prefer. Then check your work using a different method.

Compare your answers in the Appendix.

Read

Sometimes stores offer a second discount on an already marked-down item. In this case, you can't work out the answer the same as figuring out the taxes in "Example 1." Find out the difference by reading "Example 2: Percent of a Percent" on page 147 of the textbook.

Self-Check

SC 2. Answer the "Show You Know" question on page 148 of the textbook to practise what you have learned. Assume the regular price on the item is $100.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 148 of the textbook. Transfer the important ideas you have gained to your foldable.

Connect

Now that your skills at working with combined percents are stronger, how about applying them to the world around you? Put your newly developed mathematical skills to work by completing the Self-Check questions below.

Self-Check

SC 3. Complete "Communicate the Ideas" question 2 and "Check Your Understanding" questions 4, 6, 8, 10, and 12 on pages 148 and 149 of the textbook. If you are having difficulty with any answers, discuss them with your partner or your teacher until you are sure you know how to do that type of question accurately. It's very smart to ask questions.

Compare your answers in the Appendix.

Extra Practice

If you were able to accurately answer at least five of the above questions, and you feel you have a solid understanding of how to work problems with combined percents, go on to the assignment.

If you need a bit more practice with the concepts and ideas of this lesson, you may complete "Check Your Understanding" questions 5, 7, 9, 11, and 13 on pages 148 and 149 in the textbook. When you finish each question, check your work using the shortened answers given on pages 487 and 488 at the back of the textbook. Make sure to contact your teacher if you still have questions.

Assignment

To inform your teacher of your understanding of the ideas in this lesson, this question set will need to be completed and submitted to your teacher.

Go to the Unit 5 Assignment Booklet, and complete "Unit 5: Lesson 4 Question Set."

Try This

TT 6. In the previous lesson you began to research some ways to conserve water and to discuss those using percent, which will help you to prepare for your Unit Problem. The "Math Link" at the bottom of page 149 will give you additional practice discussing water conservation using percent. Think about how each of the examples given may help you with your Unit Problem assignment. Complete the calculations indicated.

Going Beyond

We often hear in the news about stock markets rising and falling. Stock markets go up and then down all the time. The analysts often speak of the changes in terms of percents. Suppose in one year the Toronto Stock Exchange (TSX) was at 14 000 and went down 40%; and then in the next year, it went up 20%. If in the following year the TSX went up another 30%, what would the value be? What is the percent change in value over the three years? Do you see how multiple percents can sometimes be misleading?

Compare your answer in the Appendix.

Lesson Summary

This lesson dealt with how to solve problems involving combined percents. There were two different categories discussed.

The first type of problem involves taking several percents of the same amount, such as finding the total cost of an item when GST and PST must be added. Three different methods to find the answer were shown:

- Calculate the amount for each of the percents and add them to the price.
- Add the percents together, find the amount for the combined percents, and add them to the price.
- Add the sum of the percents to 100% and find the total cost in one multiplication.

Using any of the three methods will yield accurate results.

The second type of problem involves taking percents of percents. This calculation is often used in sale items, where a discounted item has a further percent reduction. It is also used with a series of percent increases or decreases over a number of years, such as in population growth or compound interest. In these types of situations, the numerical value for the first percent is calculated. Then the next percent is taken off the newly calculated value.

For instance, suppose an $80 pair of jeans was on sale at 25% off, and, for a special one-day promotion, there was a 20% discount off the sale price. To find the discounted price, you would do the following:

- Calculate the price after the initial 25% reduction.

  0.25 x 80 = 20

  So subtract $20 from the $80 price to get $60.

- Find the price after the 20% discount of the first sale price.

  0.20 x 60 = 12

  So subtract $12 from the first sale price of $60 to get $48.

The price of the designer jeans during the promotion will be $48.

The two different types of problems each require a different strategy. If the various percents are of the same number, you would use the strategy for the first type of problem. If the problem is a percent of a percent, you would use the method for the second type of problem.

Unit 5: Percent, Ratio, and Rate

Lesson 5: Understanding Two-Term and Three-Term Ratios

Get Focused

© Gary Alvis/iStockphoto

Water conservation, the topic for your Unit 5 Problem, is an important concern for many people. In south-eastern Alberta, water from wells is depleting an underground water source known as the Milk River aquifer. Despite some farm homes being abandoned, as shown in the picture, there has been a 2 : 1 increase in wells using the aquifer in the last 40 years.

Federal, provincial, and municipal governments have formed an advisory committee to deal with the problem and have provided funding to deal with the situation. The ratio of funding provided by federal, provincial, and municipal governments, respectively, is approximately 3 : 3 : 1. Of particular concern are flowing wells, which do not need a windmill, like the one pictured above, to pump water to the surface, but instead waste water by running it out on top of the ground. So far, about one in four of the wells scheduled to be closed have been sealed, saving over 13 million litres of groundwater per year. This project won the group an Emerald Award, which recognizes excellent environmental initiatives in Alberta.

So far in Unit 5 you have mostly worked with percents to describe situations and solve problems. In this lesson you will learn to work with ratios in making comparisons. You will use

two-term and three-term ratios and convert them to fractions and percents. As you progress through the concepts, you will develop skill in solving problems involving ratios.

This lesson will help you answer the following critical question: How can you write equivalent ratios that apply to real-life situations?

You will need a ruler and a calculator in this lesson.

Assignments

Your assignments will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 5 Question Set

Explore

You may have taken a picture to a photo lab for an enlargement or enlarged an image yourself on a computer. How can you describe the amount of enlargement using a **ratio**?

ratio: a comparison between quantities measured in the same units

The ratio may be written in words as a to b, or in symbols as $a : b$ (It is pronounced as "a to b").

Try This

TT 1. Give that situation a try by working through "Explore the Math" questions 1, 2, and 3 on pages 46 and 47 of the textbook. Assume Kate is asking how much larger the second image is when compared to the smaller image.

Working with a partner in this activity will help you learn these math concepts. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

Note: In order to answer question 2, it may be helpful to know what the term *multiplier* means. A multiplier is the number used to multiply all of the measurements of an object in order to enlarge it. For example, to triple an image, multiply all of its measurements by 3. (The number 3 is the multiplier.)

To demonstrate your understanding, complete the following Try This questions. If possible, discuss the questions with a partner as you work through them.

TT 2. Jeremy researches and finds that an American automobile has a height of 160 cm. If a hobby store model of that car has a height of 5.0 cm when fully assembled, what is the size ratio of the model to the original car?

TT 3. A two-cycle engine in a motocross bike must have oil mixed with the gasoline in order to run properly. If the directions for mixing the fuel say to add 95 mL of oil to 3800 mL of gasoline, what is the gasoline to oil mixture ratio?

TT 4. A cookie recipe that Sarah is reading directs her to add $3\frac{1}{2}$ cups of sugar to $4\frac{1}{2}$ cups of flour. That makes a very large batch of cookies. What is the ratio of sugar to flour in this recipe expressed in the smallest whole-number ratio?

TT 5. Rupinder must organize a banquet for a wedding with 400 guests. She knows she will need 25 people to help serve the guests. What will be the ratio of servers to guests at the banquet?

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Find at least two more situations in your current life, besides the ones in the Try This questions you just answered, where ratios could be used to describe something. Make up a question about each situation. Discuss each question and the solution to it with your partner, if possible. Then post your questions, along with your responses to the TT questions, onto the discussion board. Finally, respond to two of the postings by stating whether or not you think the answers are accurate and why you think that.

Read

Ratios are an efficient and effective way to make comparisons. There are different ways to represent ratios. As such, it is important for you to be familiar with the different methods—not just so you can understand what others are trying to communicate, but also so that you can accurately describe a situation to others. See how each method is used as you carefully go through the reading in "Example 1: Represent Ratios" on pages 47 and 48 of the textbook. Pay close attention to the definitions and diagrams in the margins of pages 47 and 48 of the textbook. The information in the margin notes will help to clarify what you are reading. There are several kinds of ratios you will encounter:

- **part-to-part ratios**
- **part-to-whole ratios**
- **two-term ratios**
- **three-term ratios**

Can you spot where these different kinds of ratios are referred to in the reading?

In the reading, you will also see that ratios can be represented by diagrams, words, symbols, or fractions. Where in "Example 1" was the variety of representations used?

With all those differences packed into one short example, you may need to reread and review it several times to get it all straight in your mind.

You should create a new foldable for Unit 5. You will use it for Lessons 5, 6, and 7. Directions for building a foldable are given on page 44. Now would be a good time to construct it so you can keep track of what you learn in "Example 1" while it is fresh in your mind.

part-to-part ratio: a comparison between different parts of a group

For example, "The number of bolts : nuts in the bowl is 1 : 2."

part-to-whole ratio: a comparison between one part of a group to the whole group

For example, "The number of pencils : writing instruments in the box is 3 : 7."

two-term ratio: a comparison between two quantities measured in the same units

The ratio is written as *a* to *b* or in symbols as *a* : *b* (read as "*a* to *b*"). For example, "The number of red : green candies is 2 : 5."

three-term ratio: a comparison between three quantities measured in the same units

The ratio is written as *a* to *b* to *c* or in symbols as *a* : *b* : *c* (said "*a* to *b* to *c*"). For example, "The number of red : green : white candies is 2 : 5 : 4."

Self-Check

SC 1. When you have finished reading "Example 1," answer the "Show You Know" questions on page 48 of the textbook. Use one method to complete the answers. Then check your work using a different method. If you don't understand an answer, discuss it with your partner or teacher until you do.

Compare your answers in the Appendix.

Read

Understanding the meaning of a ratio, and expressing the ratio accurately as you did in "Example 1," is only part of the picture. Find out how to apply the ratios to real situations by reading "Example 2: Apply Ratios" on pages 49 and 50 of the textbook. You will also learn that, besides being expressed as a fraction, part-to-whole ratios can also be expressed as decimals and percents. To do this, you will use skills learned in Unit 5: Lesson 2, as well as skills from previous mathematics courses.

Self-Check

SC 2. Answer the "Show You Know" questions on page 50 of the textbook to practise what you have learned. Express the ratio for two batches of trail mix by using the numbers given in the question without reducing them, not in the smallest terms as you usually should do. If something there doesn't make sense to you, discuss it with your partner or teacher until you understand the answer.

Compare your answers in the Appendix.

One idea you will need to know is how to use **equivalent ratios** to solve problems. An equivalent ratio is made by multiplying or by dividing both terms of the ratio by the same number. (This may sound familiar, as equivalent ratios are very similar to the idea of equivalent fractions.)

As an example, consider the lug nuts that hold the wheels on vehicles.

equivalent ratios: ratios in which both terms of the one ratio are multiplied or divided by the same number to produce the other ratio

For example, the ratios 2 : 5 and 8 : 20 are equivalent ratios because both terms of the first ratio are multiplied by 4 to produce the second ratio. It could also be said that both terms in the second ratio are divided by 4 to produce the first ratio.

Question

A certain car has a three-term ratio of cars to wheels to lug nuts of 1 : 4 : 20. How many lug nuts would there be in a showroom that contained six of those cars and no others?

Solution

You need the number of lug nuts for six cars, so use the ratio of cars to lug nuts, which is 1 : 20. Then write an equivalent ratio of cars to lug nuts for six cars by multiplying both terms by 6. The equivalent ratio is 6 : 120. That ratio tells us we would have 120 lug nuts on the six cars. So we could write the new equivalent three-term ratio as 6 : 24 : 120 (e.g., cars : wheels : lug nuts).

Put a note about solving ratio problems using equivalent fractions in your new foldable, constructed using the directions on page 44.

Read

Carefully study "Key Ideas" on pages 50 and 51 in the textbook. Transfer any of the major ideas from this lesson into your new foldable, as well.

Connect

To further strengthen your skills at expressing and working with ratios, complete the following Self-Check questions.

Self-Check

SC 3. Complete "Communicate the Ideas" question 1 on page 51 of the textbook.

SC 4. Complete "Check Your Understanding" questions 5, 6, 7, 9, 11, 12, 14, 15, and 19 on pages 51 through 53 of the textbook.

Compare your answers in the Appendix.

If you are having difficulty on any answers, discuss the answers with your partner or your teacher until you are sure you know how to do that type of question accurately. It's smart to ask questions when you aren't sure.

Extra Practice

If you were able to accurately answer at least eight of the previous questions, and you feel you have a solid understanding of how to work ratio problems, go on to the assignment.

If you need a bit more practice with the concepts and ideas of this lesson, you may complete questions 8, 10, 13, 16, 17, 18, and 20 from "Check Your Understanding" on pages 51 through 53 in the textbook. When you finish each question, check your work using the shortened answers given on page 482 at the back of the textbook. Make sure to contact your teacher if you still have questions after completing this additional practice.

Assignment

To inform your teacher of your understanding of the ideas in this lesson, this question set will need to be completed and submitted to your teacher. Be sure to show all your work.

Go to the Unit 5 Assignment Booklet, and complete "Unit 5: Lesson 5 Question Set."

Going Beyond

The golden rectangle is a famous special shape with a specific length to width ratio. This ratio is pleasing to the human senses, and is found in architecture, music, and nature. It is so famous it even has a Greek letter for a name, Φ (Phi). Renaissance artists called it the "divine proportion," and it is found in the Mona Lisa and other famous works of art. The golden rectangle is closely related to squares and the Pythagorean relationship you studied in Unit 1.

Find out more about the ratio and the golden rectangle by completing question 21 on page 54 of the textbook. For more information, read "Web Link" on page 54.

Compare your answers in the Appendix.

Lesson Summary

In this lesson you learned how to understand and express ratios. The ratio is read the same, whether in symbols or in words. When one reads the symbols for the ratio 3 : 5, the person reads it as "three to five."

You learned the difference between two-term and three-term ratios. Ratios can be represented by drawings, words, symbols, fractions, decimals, and percents. This lesson required you to apply skills learned earlier in this unit to convert between fractions, decimals, and percents when expressing ratios in these forms.

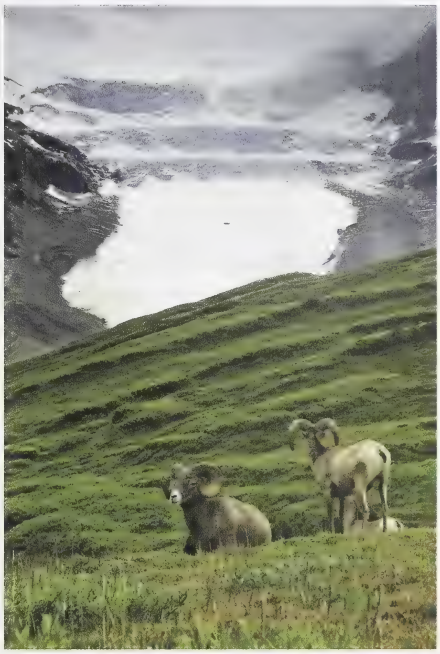
Ratio problems can often be solved using equivalent ratios. An equivalent ratio is made by multiplying or dividing both terms of the ratio by the same number.

Ratios may be thought of as an extension of the topics of fractions and percents, which were studied in previous lessons. Ratios will be used in the next lesson with proportional reasoning and for the rest of your life—both in school and out—when carefully measured comparisons are being discussed.

Lesson 6: Comparing Rates

Get Focused

A beautiful hike to Wilcox Pass above the Banff-Jasper highway has spectacular views of the Athabasca Glacier. This view, however, is changing every year as the glacier shrinks due to climate change. During the last century, the length of the glacier decreased at a rate of 12 metres per year. Alarmingly, the rate of shrinking has increased dramatically in the last decade. The thickness of the glacier is also decreasing at about 2.5 metres per year.



© sburel/iStockphoto
Two bighorn sheep are seen at Wilcox Pass near the Banff-Jasper highway.

Meltwater from glaciers is an important water source for the Prairies. In the Unit 5 Problem, there is a relationship between water conservation and the effect that glacier melting has on river flows. During the summer and fall, river flows drastically decrease, and meltwater is needed to continue to feed the rivers. The Bow River, for example, supplies water for the city of Calgary and the South Saskatchewan River. During June, the Bow has an average flow rate of 240 cubic metres/second. By August, that rate has dropped to 80 cubic metres/second. As the glaciers shrink, Albertans are becoming increasingly concerned that the rivers which supply water for population centres and agriculture will be reduced to tiny streams during the summer and fall.

When scientists and climatologists discuss the problem, they use rates of glacier shrinkage and rates of river flow. They also use rates of snowfall, rates of temperature increase, rainfall rates, and snowpack accumulation rates. This lesson will help you understand rates and solve problems involving them. By the end of this lesson, expect your skill at expressing rates and solving problems about rates to be noticeably stronger.

This lesson will help you answer the following critical questions:

- How is a rate different from a ratio?
- How can you write rates for different situations?

You will need regular and jumbo paper clips, as well as a calculator for this lesson.

Assignments

Your assessment will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 6 Question Set

Explore

In Lesson 5 you measured pictures of prairie dogs using a ruler or a tape measure and recorded the measurements in millimetres. Horses, like the one in the photograph on page 55 of the textbook, have their height measured in units called *hands*. A hand unit measurement is the width of the hand—about 10 cm. A tall horse would be 17 or 18 hands high at the base of the neck. In this activity you will be measuring objects in two different sized "paper clips" units.

You will use these units to develop a conversion rate. A **rate** compares two quantities measured in *different* units. This is different than a ratio, which you studied in Lesson 5, where the quantities are measured in the *same* units.

rate: a comparison between quantities measured in different units

The rate is usually written as a fraction, like $\frac{46 \text{ songs}}{24 \text{ hours}}$, where the units in the numerator are different than the units in the denominator.

We can talk about rates in many different ways. The heart rate is measured in beats per minute (beats/min). The rate of fuel consumption for vehicles is measured in litres per 100 km (L/100 km). The shrinkage in the length of glaciers can be measured in centimetres per day (cm/d) or metres per year (m/y or m/a where "a" represents year, as in "annual").

Try This

TT 1. How about developing your own rate? Complete "Explore the Math" questions 1, 2, 3, and 4 on pages 55 and 56 of the textbook. You can download a copy of a table to record your measurements by going to the Math 8 Multimedia DVD and opening "Paper Clips Measurement Table." Be sure to read the margin notes on pages 55 and 56, and put the definitions into your foldable. If you are not in a classroom, you can measure items in your own environment.

You will need to work with a partner for this activity to help you learn these math concepts. Ask your teacher about who an appropriate partner would be. Your partner should be someone with whom you can talk over the activity as you do it, not just someone to check your work.

Each of you will need to measure some of the objects. Decide together which objects to measure and who will measure each particular object. Every object will need to be measured twice, once with small paper clips and once with the large ones. When one partner is measuring, the other partner should record the measurements. You could each use different sized clips to take measurements or you could take turns using both sizes.

To demonstrate your understanding, complete the following Try This questions. Work with a partner on these, if you can, by discussing how to mentally figure it out and working out the answers in your heads.

TT 2. An athlete measures her heart rate during vigorous exercise and finds a rate of 34 beats in 10 seconds. What would her heart rate be in beats per second?

TT 3. What would be the heart rate of the athlete in question 1 measured in beats per minute?

TT 4. When the athlete first wakes in the morning, she measures her resting heart rate at 9 beats in 10 seconds. What would be her resting heart rate in beats per second?

TT 5. What would be the resting heart rate in beats per minute for the athlete in question 3?

TT 6. A glacier in Alberta is 1200 m shorter now than it was 100 years ago. What is the rate of shrinking measured in metres per year?

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Find two situations involving school in which rates could be used as a description—for example, the number of questions worked on per day or the number of sheets of paper used per week. Make up a question about each situation. For example, Delores uses a 150-sheet binder refill pack during a 30-day month. What is the daily rate of paper use? Do the following:

- Post your best question to the discussion board.
- Post your responses to the TT questions to discussion board.
- Respond to two of the postings from other students, and post your responses to the discussion board. State whether or not you think the calculated answers are accurate and explain why.

How about determining a **unit rate** on some flight problems? How is a unit rate different than a normal rate? A normal rate of flight may be $\frac{600 \text{ km}}{30 \text{ h}}$, but the unit rate would be 20 km/h, where the second term is one unit (1 hour).

unit rate: a rate in which the second term is one unit

The rate is usually written as a fraction, such as $\frac{16 \text{ questions}}{1 \text{ hour}}$ or 16 questions/h.

Read

Give "Example 1: Determine Unit Rates" on pages 56 and 57 of the textbook a try. See how the textbook uses estimation first to arrive at an approximate answer. Then the estimate is used to check the final answer to see if the calculated unit rates are reasonable. In other words, the calculated unit rates should be close to the estimate. Be sure to carefully read the margin notes on pages 56 and 57. The margin notes provide hints that make the questions easier to solve because you will better understand how to correctly express the rates.

Self-Check

SC 1. When you have finished reading "Example 1," answer the "Show You Know" questions on page 57 of the textbook to practise what you have read. There may be several good ways to arrive at the answer mentally. If you are unsure of a method to find the unit rate, discuss it first with your partner, and then ask the teacher about it.

Compare your answers in the Appendix.

Read

What about finding the best deal when buying groceries? Get some more pointers by reading "Example 2: Compare Prices Using Unit Prices" on pages 57 and 58 of the textbook. Note how the definition of *unit price* in the margin on page 58 is slightly different than the definition for *unit rate* on page 56. Unit price can be for 100 g or 100 mL instead of for just 1 g or 1 mL because those single amounts are too small to be purchased in bulk. You can decide whether to do the calculations for a single unit of measure or for a larger size if that is more convenient. Either way is correct.

Note that the textbook does not do an estimate in "Example 2." The estimation is not necessary because each of the three rates should be somewhat close. So, if one is way different, then you would suspect that an error had been made in calculating that one.

Also, pay attention to how the textbook converts from dollars to cents to get the answer to fewer decimal places. This is accomplished by multiplying by 100 because there are 100 cents in a dollar. Litres are also converted to millilitres by multiplying by 1000 because there are 1000 millilitres in a litre. This is done so all the volumes in the problem are in the same units.

If you would like a quick and interesting refresher on converting metric units, go to the Math 8 Multimedia DVD and open "Metric Conversion."

Self-Check

SC 2. When you have finished reading "Example 2," answer the "Show You Know" question on the bottom of page 58. If you are having difficulties with a question, discuss your challenges with a partner, if you can, or with your teacher.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 59 of the textbook. Put the rest of the important ideas from this lesson into your foldable.

Connect

Now that your skills in using rates are stronger, how about applying them to practical situations in the world around you? As you complete the Self-Check questions below, you will determine some unit rates and some unit prices as you did in the previous examples. Then you will get to choose which rates to use in solving a variety of other problems.

Self-Check

SC 3. Complete "Communicate the Ideas" questions 1 and 2 on pages 59 and 60 of the textbook.

SC 4. Complete "Check Your Understanding" questions 4, 6, 7, 8, 10, 12, and 14 on pages 60 and 61 of the textbook. Refer back to the examples if you need to as you do them.

Compare your answers in the Appendix.

If you can't figure out a question or need help with some part of one, be sure to ask your teacher about it.

Try This

In preparation for your Unit 5 Problem, you will need to do some calculations about glacier melting. The Peyto Glacier in Canada's Banff National Park retreated 2700 m between 1896 and 2002. Seventy percent of its ice mass is gone.

TT 7. What is the unit rate of retreat of this glacier?

TT 8. What is the average percent loss per year for this glacier?

TT 9. Compare your answer to TT 7 with your calculations for the Peyto Glacier in question 12 on page 61 in the textbook. Why are the rates different?

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Discuss your answers to the previous three questions with your partner, if possible. Then post your responses to these TT questions onto the discussion board. Finally, respond to two of the postings by stating whether or not you think their answers are accurate and why you think that. Did you get any new insights into question 3 from other students?

Extra Practice

If you were able to answer at least seven of the SC 3 questions accurately without having to look at your notes or change your calculations, you may go on to the following assignment. If you need a bit more practice, complete "Check Your Understanding" questions 5, 9, 11, 12, 15, and 16 in the textbook. When you finish each question, check your work using the answers given on pages 482 and 483 at the back of the textbook. Be sure to ask your teacher about a question if it is giving you difficulty.

Assignment

To let your teacher know how well you understand the ideas set out in this lesson, complete this question set and submit it to your teacher. Show all of your work.

Go to the Unit 5 Assignment Booklet and complete "Unit 5: Lesson 6 Question Set."

Lesson Summary

In this lesson you learned to express rates in both words and symbols. The rate of glacier shrinking could be stated as 12 metres per year or, in symbols, as 12 m/y.

You found that rates are different than ratios. A rate is used to compare quantities measured in two different units, such as metres and years; meanwhile, a ratio is used to compare quantities measured in the same units.

Some rates are called *unit rates* because the second term is 1. Common speed limit signs of 50 km/h are expressed in unit rates because the second term refers to "one hour." The price of gasoline is a unit rate because when written as 79.9¢/L, the second term refers to "one litre" of gasoline.

Unit rates can be used as conversion rates. A rate can be converted to a unit rate by dividing both terms by the number in the second term.

For example, a speed of 400 km in 8 h, $\frac{400\text{ km}}{8\text{ h}}$, is converted to a unit rate by dividing 400 km by 8 and dividing 8 h by 8 to get $\frac{50\text{ km}}{1\text{ h}}$. The unit rate is stated as "50 kilometres per hour" or expressed in symbols as 50 km/h.

The concepts in this lesson will be used in Lesson 7 to solve proportional reasoning problems. You will also need to call upon your understanding of ratios, fractions, decimals, and percents gained in previous lessons.

Unit 5: Percent, Ratio, and Rate

Lesson 7: Using Proportional Reasoning

Get Focused

© fjvsoares/iStockphoto

As this girl looks at the meltwater surrounding the toe of a glacier, there seems to be plenty of ice and plenty of water. In western Canada this may not be true for long. Scientists are trying to predict what the future holds. Just across Alberta's southern border, Glacier National Park in Montana has only 50 glaciers of the 150 glaciers it once had. All of them are expected to disappear in the next 30 years. Will global climate change cause a similar disappearance in Alberta? Between 1896 and 2002, 70% of the Peyto Glacier's ice mass in Canada's Banff National Park has gone—turned to meltwater in the South Saskatchewan River basin. When will the Peyto Glacier disappear and its flow of meltwater dry up?

To answer the question in the paragraph above, which is similar to questions you will solve for your Unit 5 Project, it will be necessary to make calculations involving rate and proportional reasoning. The concepts you will discover in this last lesson of Unit 5 will help you solve proportional reasoning problems. These problems may involve either rates or ratios. You will learn to find the answers for both kinds. You will be able to apply the skills in finding equivalent ratios and equivalent rates that you developed in Lessons 5 and 6 as you solve those problems.

This lesson will help you answer the following critical question: How can you use proportional reasoning to solve problems involving rates and ratios?

You will need a calculator for this lesson.

Assignments

Your assessment will consist of the following:

- posting and responding to the discussion board
- adding to your Math 8 folder
- completing Unit 5: Lesson 7 Question Set

Explore

Watching a skilled snowboarder working a half-pipe can be exciting. Trying to photograph one is even more engaging. It requires some ability to understand and use rates, ratios, and proportions.

Watch and Listen

To see rates, ratios, and proportions that are used in sports photography, watch a winter sports photographer and his subjects in action by going to the Math 8 Multimedia DVD and opening "Exploring Rate, Ratio and Proportion."

When you have finished watching the video, use your computer mouse to choose the "Interactive" button to the left of the video.

You will be shown two ratios. The object is to decide whether you can enlarge the original ratio to match the target ratio. After looking over the two ratios given in the interactive section, you should do the following:

- Click the circle (1x, 2x, 3x, 4x, or 5x) that indicates the closest number by which both terms of the original ratio should be multiplied to obtain the target ratio.

- Drag either the = or ≠ sign into the box in-between the original and target ratios to indicate whether or not the two ratios are equivalent.

- Click "Check" to see if your choice matches the situation. A check mark appears if your choice was accurate, and an "X" appears if the answer was not correct. Then watch the expansion of the image to try and match the target. Read the contents of the box at the top of the display for a message appropriate to your choice.

- Click "Next" to go on to another ratio.

After you have completed four ratio questions, choose "New" at the bottom of the screen to try four other target ratios. When you have completed all four, decide whether you can do the equivalent ratios correctly or if you could use more practice. If you need to practise some more, click "New" and continue the process until you are more confident about your abilities to choose correctly.

Try This

TT 1. To demonstrate your understanding, complete the print activity that goes along with the interactive applet you just finished. Work with a partner on these questions, if you can, by discussing how to figure out the answers before you insert them.

Use your computer mouse to choose the "Word" or "PDF" button to the left of the video under "Print Activities." Choose "Word" if your computer is a PC and "PDF" if you are using a Mac system.

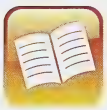
If you are submitting materials on paper, print out the exercise and answer the questions.

If you want to answer digitally, save a copy of the file to your computer. Use a different coloured font so your teacher can clearly find your answers, and insert your answers directly into the document. For questions 4 and 6, you can bold your choice and change the colour. For question 5, you may use the # sign instead of a checkmark.

Place a copy of your answers in your Math 8 course folder.

Discuss and Share

Make a summary sheet telling what you were supposed to have learned from this activity, and post your response onto the discussion board. Finally, respond to two of the postings by stating whether you agree with the summaries. You can make suggestions for improving the summaries if you feel information was missed or the description was unclear.



Read

Put your skill at writing equivalent ratios and rates to work as you study "Example 1: Solve a Rate Problem Using Proportional Reasoning" on pages 64 and 65 of the textbook. Be sure to carefully read the definition for **proportion** written in the margin of page 64. That would be something to quickly put into your foldable. Notice how the definition applies to both ratios and rates.

proportion: a relationship stating that two ratios or two rates are equal

It may be written as fractions such as $\frac{14\text{ km}}{2\text{ h}} = \frac{35\text{ km}}{5\text{ h}}$ or stated in words such as "fourteen kilometres is to two hours as thirty-five kilometres is to five hours." The two rates are equal because the terms in the second rate are 2.5 times the terms in the first rate.

Do you recognize that two methods for determining the solution are shown? The first method is to find the unit rate. Remember that the unit rate is an equivalent rate in which the denominator is 1 unit. Then the unit rate is multiplied by the number of units needed to answer the question.

The second method is to make a proportion where the equivalent rate contains a term that shows what you want to find. Here you will apply your skill at making equivalent rates by multiplying both top and bottom terms of the rate by the same number. You may have to find that number by dividing one of the terms into the other term with the same units.

Consider the following example:

$$\frac{11.58¢}{2\text{ kWh}} = \frac{?}{30\text{ kWh}}$$

Divide the 2 into the 30 and get 15. This tells you that each term in second rate is 15 times the term in the first rate. Then it is easy to get the missing term: Multiply the number of cents by 15. Notice how the "Mental Math and Estimation (ME)" on page 65 shows how to estimate the answer to check your calculations.

Discuss and Share

Consider which of the two methods you prefer and why, and post your response onto the discussion board. Then respond to other postings by telling them the insights you gained by reading their ideas.

Self-Check

SC 1. Answer the "Show You Know" question on page 65 of the textbook. Use at least two different methods to practise the strategies presented in your reading. If you are unsure of a method of finding the answer to the question, discuss it with your partner or your teacher.

Compare your answers in the Appendix.

Read

What happens when the proportion involves a ratio, where all terms are measured in the same units? See how "Example 2: Solve a Ratio Problem Using Proportional Reasoning" on pages 65 and 66 of the textbook handles it in two slightly different ways. Use "Literacy Link" in the margin on page 65 to help you understand the question. In each method, a variable is used in the proportion to make the proportion and solution more clear. In the second method, you are shown exactly which numbers to divide in order to find the multiplier. Can you use this strategy for other ratio questions?

Self-Check

SC 2. Standard television signals, VHS video tapes, and standard DVDs all have a 4 : 3 aspect ratio of width to height. If you have a standard television set that shows a picture 42-cm high, how wide will the picture be? It may help you to get started if you express the ratio as a fraction. Be sure you know what each term of the fraction represents. If you are having difficulties with this question, discuss your challenges with a partner, if you can, and then with the teacher.

Compare your answers in the Appendix.

Read

Read "Key Ideas" on page 66 of the textbook. Put the additional ideas from this lesson into your foldable.

Connect

Now that your skills in using proportional reasoning to solve problems are stronger, how about applying them to the world around you? Put your new-found mathematical strength to work by completing the Self-Check questions below. You can use these Self-Check questions as a challenge to hone your mental math skills by first estimating the answers to the questions in your head. Then use that estimate to see if your calculated answer is reasonable.

Self-Check

SC 3. Complete "Communicate the Ideas" questions 1 and 3 on page 67 of the textbook.

SC 4. Complete "Check Your Understanding" questions 4, 6, 8, 10, 11, 14, 15, 18, 20, 22, and 25 on pages 67 through 69 of the textbook. Refer back to the examples, if necessary, as you do the questions. If you can't figure out a question or need help with some part of one, be sure to ask your teacher about it.

If you need help with question 6, check the My Guide button.

My Guide

The following is a hint for question 6. You can use the unit rate of $7 per hour.

$$\frac{\$35}{5 \text{ hours}} = \frac{\$7}{1 \text{ hour}}$$

Compare your answers in the Appendix.

Extra Practice

If you were able to answer at least 10 of the SC 3 questions accurately without having to look at your notes or change your calculations, you may go on to the assignment.

If you need a bit more practice, complete "Check Your Understanding" questions 5, 7, 9, 12, 13, 16, 17, 19, 21, 23, 24, and 28 on pages 67 through 69 in the textbook. These questions will help you solidify the calculation process in your mind. When you finish each question, check your work using the answers given on page 483 at the back of the textbook. Be sure to ask your teacher if a question is giving you difficulty.

Assignment

To let your teacher know how well you understand the ideas set out in this lesson, complete this question set and submit it to your teacher.

Go to the Unit 5 Assignment Booklet and complete "Unit 5: Lesson 7 Question Set."

Lesson Summary

In this lesson you learned that a proportion is a relationship stating that two ratios or two rates are equal. You saw that it may be written as fractions such as $\frac{8\text{ km}}{2\text{ h}} = \frac{20\text{ km}}{5\text{ h}}$ or stated in words as "8 kilometres is to 2 hours as 20 kilometres is to 5 hours."

In this lesson you learned two methods of using proportional reasoning to solve problems involving rates. Remember that a *rate* is a comparison of two quantities measured in different units, such as 48 m/6 s. Use the two following methods when you are dealing with comparisons in two different units.

- The first method is to find the unit rate.
- The second method is to make a proportion where the equivalent rate contains the variable term you want to find.

A *ratio* is a comparison of two or three quantities measured in the same units, such as 20 cm : 28 cm. Use the following two methods of proportional reasoning to solve problems involving ratios, for which the units will all be the same. Making a fraction of the ratios is the first step, and writing what the denominator and numerator represents helps keep each term straight.

- The first method is put the ratios in the proportion in lowest terms.
- The second method is to use a proportion where the equivalent ratios contain the numbers from the problem.

Either method for ratios will work if applied correctly.

The concepts and skills learned in this lesson give you the final mathematical ability needed to complete the Unit 5 Problem about water conservation and glacial runoff.

Unit 5 Summary

Getting Started

When you are thirsty, there is nothing quite as refreshing as a cool glass of water. We need the water content of our bodies to be about 50% of our body mass to be healthy. Over the last few years, concerns have been raised about the decreasing supply of fresh water in our rivers, lakes, and underground aquifers. The unit problem addresses two aspects of those concerns. The first part of the problem deals with ways we can use less water in our daily lives. The second part looks at how the predicted disappearance of glaciers will affect our future water supply.

© Martin Mark Soerensen /shutterstock

Assignment

To demonstrate to your teacher some of the skills and abilities you have learned in Unit 5, there are two parts to the assignment:

Part 1

Develop a water-conservation plan that includes information about five ways to conserve fresh water. This plan will need to be presented in a creative format.

The following are helps to complete Part 1 of the project:

- To find five ways that you could conserve fresh water, look at the "Math Links" sections you completed in Lessons 3 and 4 in this unit. (These sections are found on pages 143 and 149 of the textbook.) Lesson 4 also has information in the Get Focused section at the start of the lesson and in Try This questions in the Explore section. You may also use other sources of information on water conservation if you wish.

- For each conservation measure, estimate the amount of water normally used and the expected water savings measured in volume and in percent. Record your estimated total water savings as a percent of the normal water usage. To help organize your thinking, you may wish to use a prepared table to help sort your information. It can be found on the Math 8 Multimedia DVD by opening "Data Table 1." Be sure to include units in your measurements; e.g., 78 L/day, 4500 L/year, 22%.

- Your information should be presented as a newspaper article, cartoon strip, video, song, podcast, narrated screen shots, or another creative format of your choice.

- Have a discussion with your teacher about your ideas *early* in your decision-making process to ensure your format for submission will work. Be careful to plan a presentation that you can do in the allotted time. Find out from your teacher what criteria will be used in awarding marks so that you know how to design your presentation. In other words, ask for a copy of the evaluation rubric.

Part 2

Calculate the rate of glacial melting from statistics and predict how that will influence future river flows.

© Jason Cheever /shutterstock

The Saskatchewan Glacier is located in Banff National Park. The glacier provides water for the North Saskatchewan River.

For the second part of your assignment, you will use some broad assumptions about glaciers to get a rough idea about how fast they are shrinking and how that will affect the amount of river water flowing in the future. Scientists wrestle with these same questions; however, they have access to more data so their predictions are most likely more accurate than your predictions. You are doing this exercise to demonstrate the mathematical skills you have learned.

Choose only **one** of the following glaciers, and let your teacher know which one you are working with. Use the following information:

- Athabasca Glacier: Assume the glacier has an area of 6 000 000 m^2, is 200 m thick, and that it loses 2.5 m in thickness per year.

- Peyto Glacier: Assume the glacier has an area of 12 000 000 m^2, is 95 m thick, and that it loses 2.5 m in thickness per year.

- Saskatchewan Glacier: Assume the glacier has an area of 12 000 000 m^2, is 300 m thick, and that it loses 2.5 m in thickness per year.

Then complete the following questions.

1. Calculate the number of years it will take for the glacier to disappear; e.g., the thickness of the glacier to be reduced to zero.

2. Calculate the volume of the glacier that disappears in a year. Assume, for simplicity, that the glacier is a rectangular prism.

3. Calculate the volume of water that is produced by the disappearance of the glacier ice. Assume that the volume of water is 90% of the volume of the ice.

4. Calculate the flow rate of the water from question 3 in cubic metres per second (m^3/s) if the melting took place in just 120 days of the year when it was warm enough to melt ice. There are 86 400 seconds in a day (24 hours x 60 min/h x 60 s/min).

5. Calculate what percent of the water flowing in the Bow River at Banff in August would have come from the amount of water in question 4. Assume that the average flow rate of the Bow River at Banff in August is 54 m3/s.

   If you have chosen to work with the Saskatchewan Glacier, you may have noted that the glacier actually feeds the North Saskatchewan River. Continue to use the information about the Bow River provided in this question. The Bow River is used with question 5 for simplicity. The goal is for you to practise using the math concepts you studied in this unit.

6. How significant is the shrinkage of the glacier on the flow rate of the river?

If you need help getting started with any of the calculation steps, consult My Guide for direction.

My Guide

The example calculations are for the Bow Glacier. Assume the glacier has an area of 3 200 000 m2, is 48 m thick, and that it loses 1 m in thickness per year.

Q: For question 1, how do you calculate the number of years it will take for the glacier to disappear?

A: Set up a proportionality involving the thickness of the glacier and the loss in thickness per year. Find the time from the proportionality.

Example:

$$\frac{1\text{ m}}{1\text{ y}} = \frac{48\text{ m}}{\text{t}}$$

It will take 48 years for the Bow Glacier to disappear at that rate of thinning.

Q: In question 2, how do you calculate the volume of the glacier that disappears in a year?

A: To calculate the volume, use the area and multiply by the height loss as you learned in Unit 4.

Example:

V = area of the base x height
V = 3 200 000 m^2 x 1 m
V = 3 200 000 m^3

The volume of the Bow Glacier that disappears in a year is 3 200 000 m^3.

Q: In question 3, how do you calculate the volume of water that is produced by the disappearance of the glacier ice?

A: Find 90% of the volume of the ice. Example:

90% of 3 200 000 m^3 = 0.90 x 3 200 000 m^3
0.90 x 3 200 000 m^3 = 2 880 000 m^3

The volume of water that is produced in one year by the disappearance of ice on the Bow Glacier is 2 880 000 m^3.

Q: How do you calculate the flow rate of the water from question 3 in cubic metres per second (m^3/s) if the melting took place in just 120 days of the year when it was warm enough to melt ice?

A: Set up a rate involving the amount of water and the time, and find the unit rate for 1 day. Then set up a proportion to find the unit rate for 1 second, using the fact that 1 day = 86 400 s.

Example:

$$\frac{2\ 880\ 000\ m^3}{120\ days} = \frac{24\ 000\ m^3}{1\ day}$$

$$\frac{24\ 000\ m^3}{86\ 400\ s} = \frac{0.2\overline{7}\ m^3}{1\ s}$$

The flow rate of water from the disappearance of ice on the Bow Glacier is $0.2\overline{7}\ m^3/s$.

Q: Question 5 asks you how to calculate what percent of the water flowing in the Bow River at Banff in August would have come from the amount of water in question 4.

A: Set up a proportionality in which 100% represents the flow rate of the Bow River at Banff taken from the given information. Example:

$$\frac{54\ m^3/s}{100\%} = \frac{0.2\overline{7}\ m^3/s}{p}$$

$$p \approx 0.51\%$$

The percent of the water flowing in the Bow River at Banff in August that would have come from the disappearance of ice on the Bow Glacier is about 0.51%.

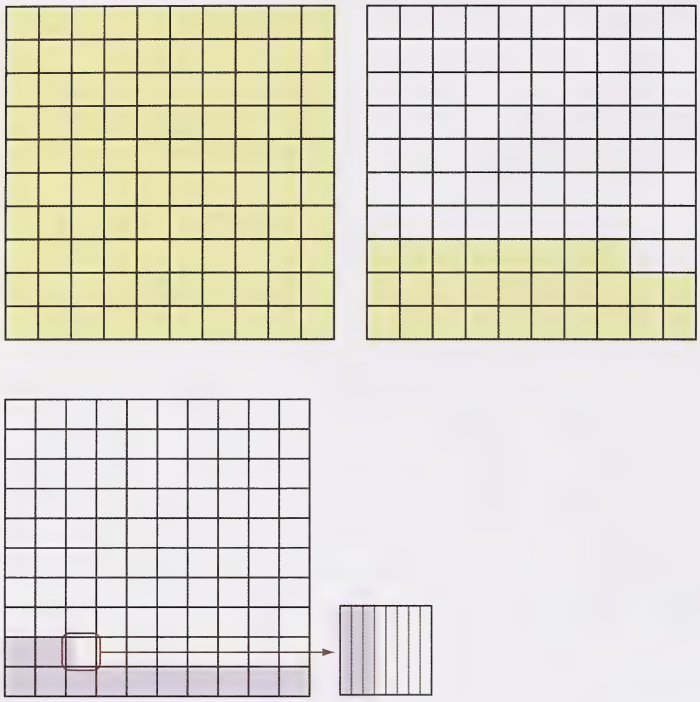
Q: In question 6, how significant is the shrinkage of the Bow Glacier on the flow rate of the Bow River?

A: The flow rate from the Bow Glacier is less than 1% of the flow rate of the Bow River, so it is not very significant.

Unit Summary

Through each lesson question, you have worked at gaining the knowledge to answer the larger Unit 5 critical question: How can you use percentages, ratios, and rates to make meaningful comparisons when investigating characteristics of living things or objects?

First you used grid paper to model percents greater than 100, fractional percents, and percents less than 1%. You then used those concepts to model fractions on a grid and learned to convert back and forth between fractions, decimals, and percents. Next, you learned how to determine percents from information supplied in a question.



When you applied your knowledge of percents to solving problems, you were led to discover ways to use mental math to arrive at an answer by breaking the problem into several simple steps. Then you learned to calculate the percent of a number by converting the percent to a decimal and multiplying the number by that decimal. As a result, you were able to calculate combined percents such as GST and PST on a purchase. As well, you also were able to take a percent of a percent to calculate the discount on already reduced sale items.

You were then able to apply those skills in representing ratios as fractions and percents. You learned to represent and describe two-term and three-term ratios and solve problems involving them. Example ratio: 20 cm : 28 cm or $\frac{20\text{ cm}}{28\text{ cm}}$.

Next you discovered that rates can also be represented as fractions, but the numerator and denominator do not have to be in the same units as they do in ratios. Example rate: $23.67 per 30 litres or $\frac{\$23.67}{30\text{ L}}$. Using this knowledge, you determined unit rates and unit prices to make comparisons and to solve problems.

Finally you were able to apply your skills in solving problems using proportional reasoning by setting up equivalent ratios and rates.

Ratio Proportionality: $\frac{20\text{ cm}}{28\text{ cm}} = \frac{100\text{ cm}}{d}$ Rate Proportionality: $\frac{20\text{ km}}{2\text{ h}} = \frac{d}{8\text{ h}}$

You learned to employ a variety of methods in solving those problems, including using unit rates and ratios expressed in lowest terms.

Unit Review

Reinforcing the concepts and skills of this unit is an important part of long-term understanding of math concepts. Take the time to look over your foldables for this unit until all the ideas are fresh in your mind. Then challenge yourself to complete some questions in the following Self-Checks that prompt you to think back on what you have learned. If any questions are too puzzling for you to figure out, be sure to contact your teacher for further assistance.

Self-Check

SC 1. Turn to "Chapter 4 Review" on pages 150 and 151.

a. Complete questions 1, 2, and 3.

b. Complete at least two questions from each of the four text sections shown:

- "4.1 Representing Percents," pages 122–129
- "4.2 Fractions, Decimals," and Percents, pages 130–137
- "4.3 Percent of a Number," pages 138–143
- "4.4 Combining Percents," pages 144–149

Check your answers after completing each section using the solutions provided on page 488 in the textbook. You may do the additional questions if you feel you need the practice.

SC 2. Turn to "Chapter 2 Review" on pages 70 and 71.

a. Complete questions 1, 2, 3, 4, and 5.

b. Complete at least three questions from each of the three text sections shown:

- "2.1 Two-Term and Three-Term Ratios," pages 46–54
- "2.2 Rates," pages 55–62
- "2.3 Proportional Reasoning," pages 63–69

Check your answers after completing each section using the solutions provided on pages 483 and 484 in the textbook. You may do the additional questions if you feel you need the practice.

Compare your answers in the Appendix.

Are You Ready?

You've done a lot of work to reach this point in the unit, haven't you? But are you ready to strut your stuff—that is, show your mastery of the new concepts and skills? Are you really ready to take the challenge of a unit test?

Complete the following Self-Check questions.

Self-Check

There are two chapters in this unit, so there are two practice tests in the textbook for you to complete. Ask your teacher whether you should do the odd or the even questions on both tests.

SC 3. Turn to pages 152 and 153, and complete the designated questions in "Practice 4 Test."

Compare your answers in the Appendix.

SC 4. Turn to pages 72 and 73, and complete the designated questions in "Practice 2 Test."

Compare your answers in the Appendix.

Assignment

Check with your teacher about a Unit 5 test.

Appendix

Lesson 1

SC 1.

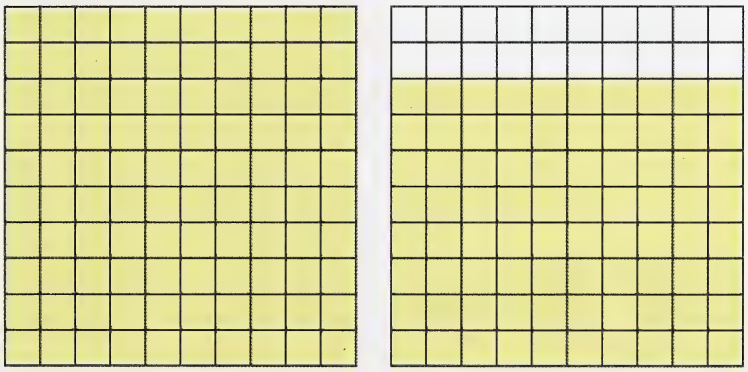
a. 248%

b. $\frac{1}{4}\%$ (or 0.25%)

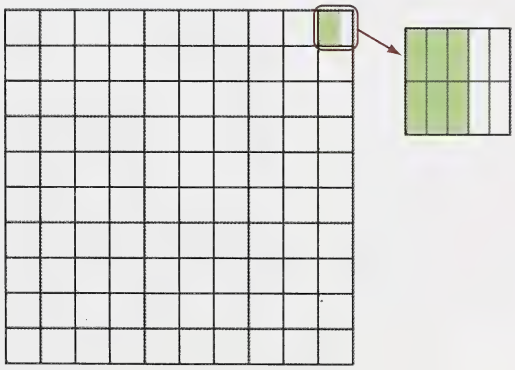
c. 74.8% (or $74\frac{8}{10}\%$)

SC 2.

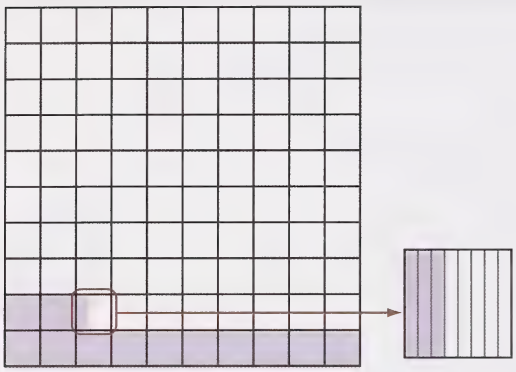
a.



b.



c.



SC 3.

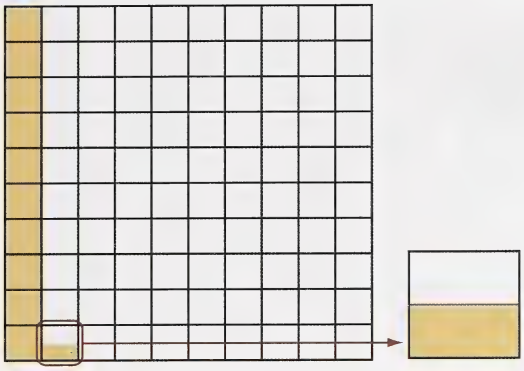
4.a. The 112 squares coloured represents 112%.

b. The two small squares out of ten that are filled in the one square represents 0.2%.

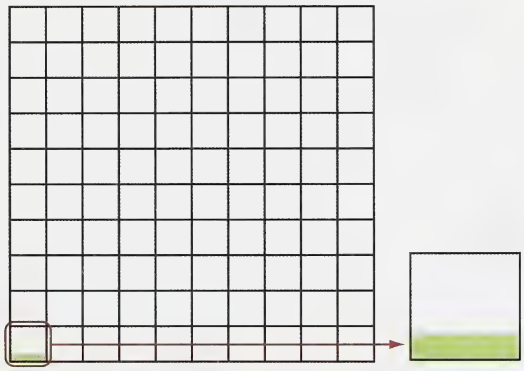
c. There are 85 complete squares coloured and one out of three small squares in the 86th square. This represents $85\frac{1}{3}\%$.

6.a.

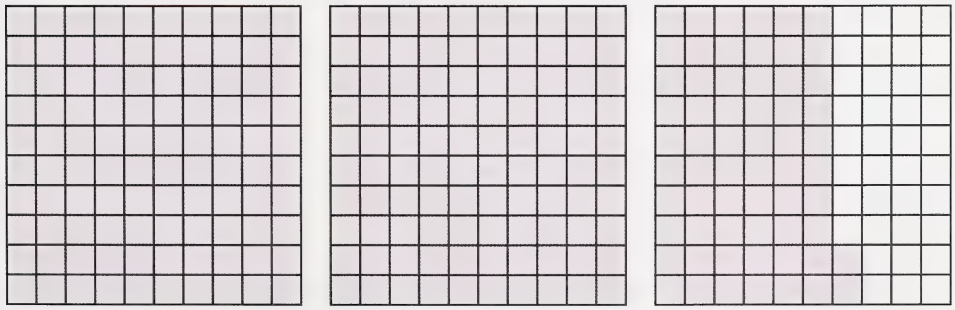
b.



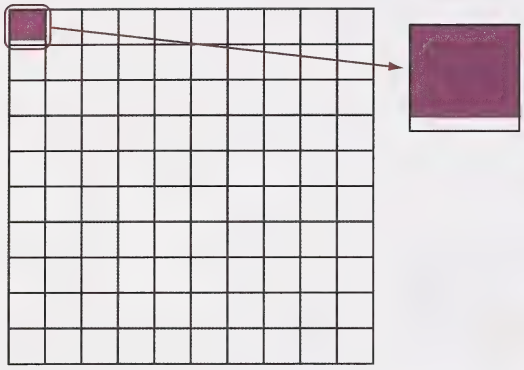
c.



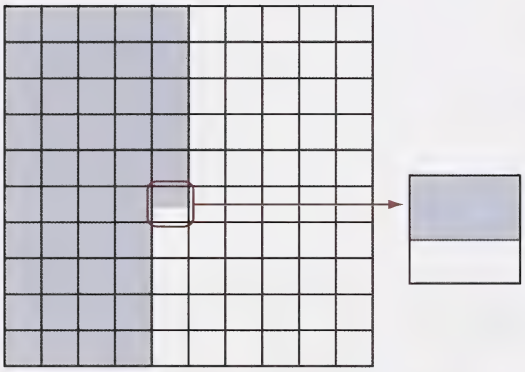
d.



e.



f.



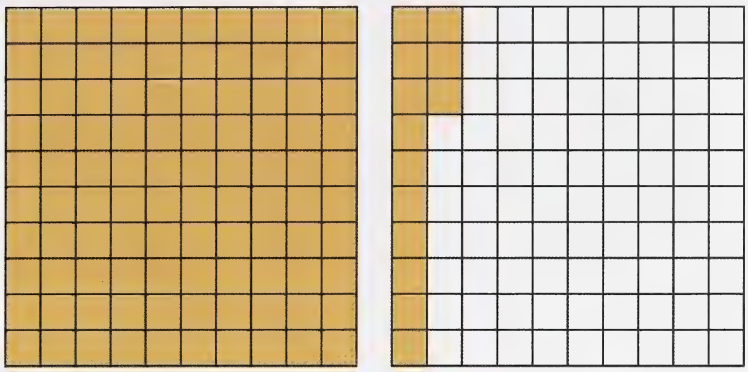
8.a. It takes three hundred grids to show 300%. (Note that this does not mean 300 grids. It means three 10 by 10 grids.)

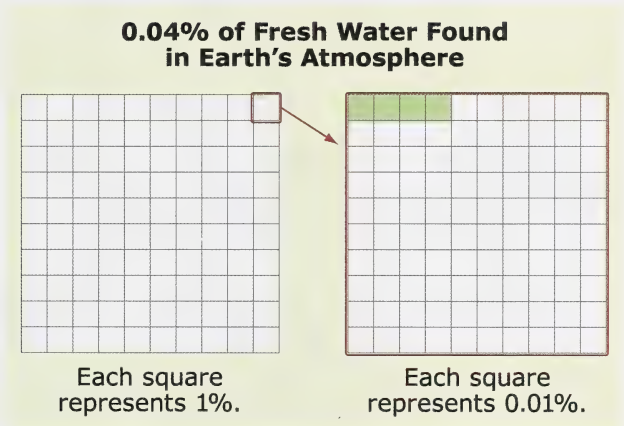
b. It takes five hundred grids to show 466%. (Note that this does not mean 500 grids. It means five 10 by 10 grids.)

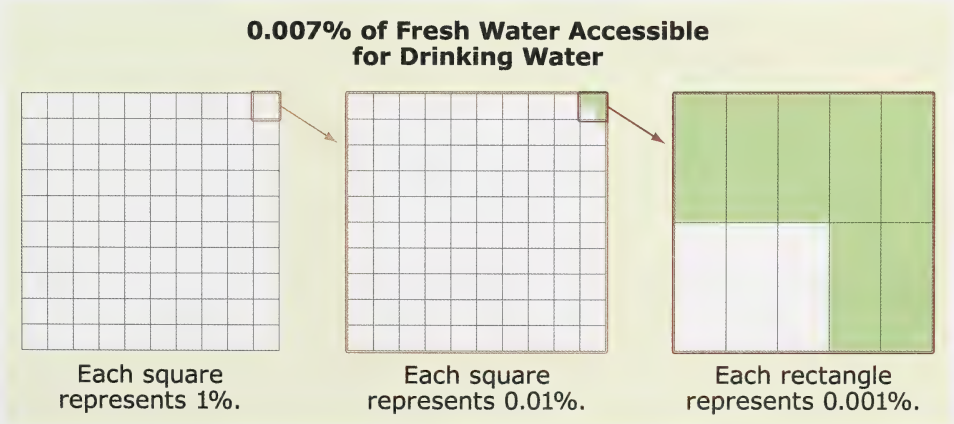
c. It takes 12 hundred grids to show 1200%. (Note that this does not mean 1200 grids. It means twelve 10 by 10 grids.)

10. The water scientist might use very small percentages since it takes very little of a chemical, for example, to cause dangerous pollution. It might be 10s of parts in billions of parts of water.

11. The following hundreds grids relate the area of Alberta to that of Saskatchewan.






0.007% of Fresh Water Accessible for Drinking Water
Each square represents 1%.
Each square represents 0.01%.
Each rectangle represents 0.001%.

Lesson 2

SC 1.

a. $\frac{3}{40} = 0.075 = 7.5\%$

b. $\frac{171}{300} = 0.57 = 57\%$

c. $\frac{88}{50} = 1.76 = 176\%$

SC 2.

a. $0.0064 = 0.64\% = \frac{64}{10\,000} = \frac{4}{625}$

b. $0.268 = 26.8\% = \frac{268}{1000} = \frac{67}{250}$

c. $5.98 = 598\% = \frac{598}{100} = \frac{299}{50}$

SC 3.

a. $750\% = 7.5 = \frac{750}{100} = \frac{15}{2}$

b. $0.3\% = 0.003 = \frac{3}{1000}$

c. $12\frac{3}{4}\% = 0.1275 = \frac{1275}{10\,000} = \frac{51}{400}$

SC 4.

a. $\frac{75}{10} = 7.5 = 750\%$

b. $\frac{75}{40\,000} = 0.001875 = 0.1875\%$

SC 5.

1. Kaitlyn: $0.003 = \frac{3}{1000} \neq \frac{3}{100} = 3\%$

   Jordan: $0.003 = \frac{3}{1000} = \frac{0.3}{100} = 0.3\%$

2. The third number from the left, 250%, is different than the others.

   $\frac{12}{5} = \frac{240}{100} = 2.4 = 240\%$

   $2.4 = \frac{240}{100} = \frac{12}{5} = 240\%$

   $250\% = \frac{250}{100} = \frac{5}{2} = 2.5$

   $\frac{60}{25} = \frac{12}{5} = \frac{240}{100} = 2.4 = 240\%$

4. a. $\frac{1}{250} = 0.004 = 0.4\%$

   b. $\frac{81}{200} = 0.405 = 40.5\%$

   c. $\frac{7}{5} = 1.40 = 140\%$

6. a. $0.0072 = \frac{72}{10\,000} = \frac{0.72}{100} = 0.72\%$

   b. $0.548 = \frac{548}{1000} = \frac{54.8}{100} = 54.8\%$

   c. $3.45 = \frac{345}{100} = 345\%$

8. a. $248\% = \frac{248}{100} = 2.48$

   b. $0.56\% = \frac{56}{10\,000} = 0.0056$

   c. $75\frac{3}{4}\% = \frac{75.75}{100} = 0.7575$

11.

a. $\frac{17}{25} = \frac{68}{100} = 0.68 = 68\%$

b. $\frac{9}{24} = \frac{3}{8} = \frac{37.5}{100} = 37.5\%$

13. The current price is 2000% of the price several years ago.

$$\frac{\text{current price}}{\text{original price}} = \frac{\$200}{\$10} = \frac{2000}{100} = 2000\%$$

15. The numbers in ascending order are 145%, 1.32, 0.65, 33.5%, 5/8%, and 0.6%.

16. To the nearest tenth of a percent is 0.4%. As a repeating decimal this is $0.00\overline{4}$, and as a fraction in lowest terms this is $\frac{1}{225}$.

$\frac{12}{2700} = \frac{1}{225} = 0.00\overline{4} = 0.\overline{4}\%$

18. The completed chart should look like the following.

Species	Number	Percent of Total	Fraction of Total	Decimal Equivalent
Chinook	143	$\frac{143}{267} \approx 53.59\%$	$\frac{143}{267}$	$\frac{143}{267} \approx 0.535\,580\,524\,3$
Coho	122	$\frac{122}{267} \approx 45.69\%$	$\frac{122}{267}$	$\frac{122}{267} \approx 0.456\,928\,239$
Steelhead	2	$\frac{2}{267} \approx 0.749\%$	$\frac{2}{267}$	$\frac{2}{267} \approx 0.007\,490\,636\,7$
Total	267			

SC 6.a. glaciers: 68.9% = 0.689 = $\frac{689}{1000}$

groundwater: 30.8% = 0.308 = $\frac{308}{1000}$

lakes and rivers: 0.3% = 0.003 = $\frac{3}{1000}$

b. If the glaciers melt into the ocean, the total amount of Earth's fresh water will decrease. As the glaciers melt, the percentage of Earth's fresh water in glaciers will decrease. If the amount of fresh water in the ground and in lakes and rivers stays the same, the percents of Earth's fresh water in groundwater and in lakes and rivers will rise. That is because the groundwater and the lakes and rivers will be a larger fraction of the smaller total fresh water.

Percent	Decimal	Fraction
f. 31.25	0.3125	$\frac{5}{16}$
g. 15.625	0.15625	$\frac{5}{32}$
h. 7.8125	0.078125	$\frac{5}{64}$

Lesson 3

SC 1.

a. 100% of 10 = 10
300% of 10 = 30
50% of 10 = $\frac{1}{2}$ of 10 = 5
30 + 5 = 35

So, 350% of \$10 = \$35.

b. You can find 1% by dividing by 100, since 100% ÷ 100 = 1%.
So, 1% of 5000 = 50.

5000 ÷ 100 = 50

To go from 1% to 0.1%, you divide by 10.

50 ÷ 10 = 5
0.1% of 5000 = 5
So, 0.1% of \$5000 = \$5.

c. 100% of 20 000 is 20 000
1% of 20 000 = 200
20 000 ÷ 100

2% of 20 000 = 400
200 x 2

$\frac{1}{10}$ % of 20 000 = $\frac{1}{10}$ of 1% = 20
200 ÷ 10
400 + 20 = 420

So, $2\frac{1}{10}$ % of $20 000 is $420.

SC 2.

a. 160% of 53.27 = 1.60 x 53.27
1.60 x 53.27 = 85.232
So, 160% of $53.27 is approximately $85.23.

Did you notice how the answer ($85.23) is larger than the number you started with ($53.27)? This is a great way for a quick mental check to see if you are on the right track. The percentage 160% is greater than 100%, so the answer should be larger!

b. $\frac{3}{4}$ % of 135 = ?
3 ÷ 4 = 0.75
0.75% of 135 = 0.0075 x 135

(The percent was changed to a decimal by dividing by 100, which is easily done by moving the decimal two places to the left.)

0.0075 x 135 = 1.0125
So, $\frac{3}{4}$ % of 135 is 1.0125.

The answer is not rounded because it is not in dollars.

c.

Pencil and Calculator Method	Mental Math Calculation
$55\frac{8}{10}$ % of 500 = 55.8% of 500 55.8% of 500 = 0.558 x 500 0.558 x 500 = 279 So, $55\frac{8}{10}$ % of 500 = 279.	$55\frac{8}{10}$ % of 500 = ? 50% of 500 = 250 5% of 500 = 25 1% of 500 = 5 0.8 of 5 = 4 250 + 25 + 4 = 279

SC 3.

1.a. The percent 300% divided by 100 is 3, so you calculate 3 times 40.

b. The percent 0.5% divided by 100 is 0.005, so you calculate 0.005 times 120.

c. The percent 10.5% divided by 100 is 0.105, so you calculate 0.105 time 80.

3.a. 300% of 2000 is 6000.

b. $1\frac{1}{4}$% of 60 is 0.75.

c. 0.1% of 40 is 0.04.

5.a. $$\frac{2}{5} \div 100 = \frac{2}{500}$$
$$\frac{2}{500} \times 32.5 = 0.13$$

b. $$15\frac{1}{4} \div 100 = \frac{15 \times 4 + 1}{400}$$
$$\frac{61}{400} \times 950 = 144.875$$

c. $$175 \div 100 = \frac{175}{100}$$
$$\frac{175}{100} \times \$125.50 = \$219.625$$
Since this is a money question, you might want to round the answer to \$219.63.

7.a. Your chances of winning with one ticket is 0.5% since $\frac{1}{200} = \frac{\frac{1}{2}}{100}$.

b. To have a 2.5% chance of winning, you would need to buy five tickets since $\frac{2\frac{1}{2}}{100} = \frac{5}{200}$.

9. You can find that Mount Logan is 5958 m tall, to the nearest metre, using these calculations.

Mount Logan = 159% Mount Columbia

Mount Logan = $\frac{159}{100} \times 3747$ m

Mount Logan = 5957.73 m

11. Using the following calculations, you see that the area of Manitoba is about 649 004 km^2.

area Manitoba = $6\frac{1}{2}$% area Canada

area Manitoba = $\frac{6 \times 2 + 1}{200} \times 9\,984\,670$ km^2

area Manitoba $\approx$ 649 003.55 km^2

13.a. The word *commission* means "the amount the seller pays to an agent to sell their house."

b. The following calculations show that the real estate agent will receive $18 700 as the commission for selling the $345 000 house.

$$\$345\,000 = \$200\,000 + \$145\,000$$

$$5\% \text{ of } \$200\,000 = \frac{5}{100} \times \$200\,000$$

$$= \$10\,000$$

$$6\% \text{ of } \$145\,000 = \frac{6}{100} \times \$145\,000$$

$$= \$8700$$

$$\$10\,000 + \$8700 = \$18\,700$$

Lesson 4

SC 1. The total cost of the backpack is $38.85. You should use one method to find the answer and another method to check it. Solutions for each of the methods follow:

Method 1

GST: 5% of $35 is $1.75 (0.05 x 35 = 1.75)
PST: 6% of $35 is $2.10 (0.06 x 35 = 2.10)
total tax: $1.75 + $2.10 = $3.85
total cost: $35.00 + $3.85 = $38.85

Method 2

GST + PST = 5% + 6% = 11%
total tax: 11% of $35.00 is $3.85 (0.11 x 35.00 = 3.85)
total cost: $35.00 + $3.85 = $38.85

Method 3

price + GST + PST = 100% + 5% + 6% = 111%
total cost: 111% of $35.00 is $38.85 (1.11 x 35.00 = 38.85)

SC 2. For Store A, the discount is 50% of $100 or $50.

For Store B, the first discount is 25% of $100 or $25. That gives a discounted price of $75. The second discount is 25% of $75 or $18.75. Therefore, the total discount at Store B is $25 + $18.75, or $43.75.

Store A is the place for a better buy.

SC 3.

2. First, you add the tax percentages and 100% together. Then you multiply this percentage by the cost of the item.

4. The purchase price for the items was $38.04, as these calculations show.

$$2 \times \$4.99 + \$3.99 + \$19.99 = \$33.96$$
$$100\% + 5\% + 7\% = 112\%$$
$$112\% \text{ of } \$33.96 = \frac{112}{100} \times \$33.96$$
$$= \$38.0352$$

6. The CD player would cost $38.25 in the second week, as these calculations show.

$$(100\% - 50\%) \text{ of } \$85.00 = \$42.50$$
$$(100\% - 10\%) \text{ of } \$42.50 = \$38.25$$

8.

Item Purchased	Price	Total Tax	Total Cost
a. Boots	$119.99	5% ($6.00)	$125.99
b. Pants	$89.99	5% ($4.50)	$94.99
c. Gloves	$39.99	5% ($2.00)	$41.99
d. Helmet	189.99	5% ($9.50)	$199.49

10. The total cost of the four tires will be $362.10.

$$4\times\ \$85\times(100\%+5\%+1.5\%)=\$340\times106.5\%$$
$$=\$362.10$$

12.a. The total distance of the race was 51.5 km.

The swim was $\frac{1.5}{51.5}\times100\%\approx2.9\%$ of the race.

The bike ride was $\frac{40}{51.5}\times100\%\approx77.7\%$ of the race.

The run was $\frac{10}{51.5}\times100\%\approx19.4\%$ of the race.

b. 77.7%+19.4% = 97.1% of the race was spent on land.

After the first year, the TSX was 60% of the previous year.

0.60 x 14 000 = 8400

At the end of the next year, it was 120% of that value.

1.20 x 8400 = 10 080

After the following year, it was 130% of that value.

1.30 x 10 080 = 13 104

The actual change is 14 000 – 13 104, or a loss of 896. The percent change is calculated as follows:

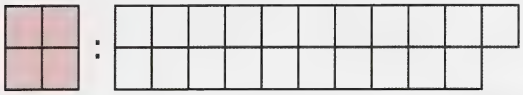
$$-\frac{896}{14000}\text{ x }100\%=-6.4\%$$

Multiple percents can be deceptive because there is a loss of 40% and then the successive gains of 20% and 30% make it sound like there is an overall gain of 10% in the three years. However, there is still a loss.

Lesson 5

SC 1.

a. The ratio of red tiles to total tiles can be expressed as 4 to 21; 4 : 21; $\frac{4}{21}$; or in a diagram.



Any three of the four ways shown is a complete answer.

b. The 4 : 6 ratio could represent the ratio of red tiles to black tiles.

c. The ratio of red tiles to black to white tiles is 4 : 6 : 11.

SC 2.

a. The ratio of mini pretzels to raisins is 6 : 2; 6 to 2; or $\frac{6}{2}$.

b. The ratio of roasted soy chips to sunflower seeds is 4 : 2.

c. The two batches of trail mix make 14 cups.

d. The ratio of soy chips and sunflower seeds to total trail mix is 6 : 14, which is also expressed as $\frac{6}{14} \approx 0.429 \approx 43\%$.

SC 3.

1. Janine wants to write the ratio of oranges to apples, so she must put the number of oranges before the number of apples. The correct ratio is 4 : 3.

SC 4. Check your work using the answers given on page 482 at the back of your textbook.

21.a. The 24 m by 38.9 m rectangle is the only example of a golden rectangle.

$\frac{38.9}{24} \approx 1.6208$

$\frac{120.5}{52} \approx 2.317$

$\frac{348}{56.5} \approx 6.159$

b. The 6.4 m wide golden rectangle has a length of 10.4 m.

$$\frac{x}{6.4\text{ m}} = 1.62$$
$$x = 1.62 \times 6.4\text{ m}$$
$$x = 10.368\text{ m}$$

Lesson 6

SC 1.

a.

estimate: $\frac{140\text{ m}}{20\text{ s}} = 7\text{ m/s}$ calculated rate: $\frac{150\text{ m}}{25\text{ s}} = 6\text{ m/s}$

Brandon's speed is 6 m/s. The estimate suggests this answer is reasonable.

b.

estimate: $\frac{\$100}{10\text{ h}} = \$10\text{/h}$ calculated rate: $\frac{\$88}{8\text{ h}} = \$11\text{/h}$

Kira earns $11 per hour. The estimate suggests this answer is reasonable.

c.

estimate: $\frac{\$10}{5\text{ cans}} = \$2\text{/can}$ calculated rate: $\frac{\$9}{5\text{ cans}} = \$1.80\text{/can}$

The cat food costs $1.80 per can. The estimate suggests this answer is reasonable.

SC 2. Change the dollars to cents by multiplying by 100. For the large size, change the 1.7 L to 1700 mL by multiplying by 1000.

small size: $\frac{344\text{ cents}}{425\text{ mL}} = 0.81¢\text{/mL}$

medium size: $\frac{629\text{ cents}}{642\text{ mL}} = 0.98¢\text{/mL}$

large size: $\frac{1549\text{ cents}}{1700\text{ mL}} = 0.91¢\text{/mL}$

The small size is the best buy because it costs the least per millilitre.

SC 3.

1.a. There are 12 possible answers using the information in the chart. Here are some examples:

- The ratio of birth mass of black bears to polar bears is 0.3 : 0.7.
- The ratio of birth mass of polar bears to black bears is 0.7 : 0.3.
- The ratio of 60-day mass of black bears to polar bears is 6.5 : 7.4.
- The ratio of 60-day mass of polar bears to black bears is 7.4 : 6.5.

b. A rate is a comparison between quantities measured in different units. A variety of accurate answers is possible. The following are examples:

- The growth rate of polar bears is 6.7 kg/60 days.

  $7.4 - 0.7 = 6.7$

- The growth rate of black bears is 6.2 kg/60 days.

  $6.5 - 0.3 = 6.2$

c. A variety of accurate answers is possible. The following are examples:

- The unit rate for polar bears is 0.112 kg/day.

  $6.7 \div 60 \approx 0.112$

- The unit rate for black bears is 0.103 kg/day.

  $6.2 \div 60 \approx 0.103$

2. For Brand A, $\frac{\$13.60}{8 \text{ cans}}$ is the rate. So, to get the unit rate, divide \$13.60 by 8 to get \$1.70/can.

For brand B, $\frac{\$8.75}{5 \text{ cans}}$ is the rate. So, to get the unit rate, divide \$8.75 by 5 to get \$1.75/can.

If you work out the price per can (unit price), you can compare the cost for each brand. The can with the lowest unit price costs the least per can. In this case, it would be Brand A.

SC 4. Check your work using the answers given on pages 482 and 483 at the back of your textbook.

Lesson 7

SC 1.

Method 1

Find the unit rate.

$$\frac{72 \text{ players}}{8 \text{ teams}} = \frac{9 \text{ players}}{1 \text{ team}}$$

The unit rate is 9 players/team. On two teams, there would be 2 x 9 players or 18 players.

Method 2

Use a proportion.

$$\frac{72 \text{ players}}{8 \text{ teams}} = \frac{18 \text{ players}}{2 \text{ team}}$$

Both terms in the first rate were divided by 4. On two teams, there would be 18 players.

Other ways of working the problem are possible.

SC 2.

$$\frac{\text{width}}{\text{height}} = \frac{4}{3}$$

The ratio is already in lowest terms, so making an equivalent ratio from the original is the method to use.

$$\frac{4}{3} = \frac{w}{42 \text{ cm}}$$

It does not matter whether height or width is on the top of the ratio, just as long as it is the same on both sides of the equal sign. The following ratio works just as well.

$$\frac{3}{4} = \frac{42 \text{ cm}}{w}$$

Divide to find the multiplier.

$42 \div 3 = 14$

Multiplying both terms by 14 will give an equivalent ratio.

w = 4 x 14
w = 56

The width of the picture is 56 cm.

SC 3.

1. A variety of answers is possible.

- A *ratio* is a comparison of two or three quantities measured in the same units.

  Example: 20 cm : 28 cm

- A *rate* is a comparison of two quantities measured in different units.

  Example: 48 m/6 s

- A *proportion* is an equation with two ratios or two rates that are equal.

  Example: $\frac{\$3.50}{5 \text{ bars}} = \frac{\$x}{8 \text{ bars}}$

3.a. $\frac{3 \text{ balls}}{\$1.25} = \frac{12 \text{ balls}}{\$x}$

b. 12 ÷ 3 = 4
x =4 x $1.25
x = $5.00

Twelve balls cost $5.00.

SC 4.

For questions 4, 8, 10, 11, 14, 15, and 18, check the answers page 483 of your textbook. An answer for question 6 is provided. You may want to check the hint for question 6 in the My Guide.

6. You can use the unit rate of $7 per hour.

$\frac{\$35}{5 \text{ hours}} = \frac{\$7}{1 \text{ hour}}$

Delia would earn $3 \times \$7 = \$21$ for 3 hours of babysitting.

20. The ratio of corn to rice is 3 : 2. A proportion could be $\frac{3}{2} = \frac{225 \text{ g corn}}{r}$, and the multiplier would be 75.

22.a. The student is 1.8 m tall, as these calculations show.

$$\frac{\text{tree height}}{\text{length of shadow}} = \frac{\text{student height}}{\text{length of shadow}}$$

$$\frac{15 \text{ m}}{9 \text{ m}} = \frac{x}{1.08 \text{ m}}$$

$$x = \frac{15 \text{ m}}{9 \text{ m}} \times 1.08 \text{ m}$$

$$x = 1.8 \text{ m}$$

b. The student's shadow would be 48 cm long.

$$\frac{\text{tower height}}{\text{length of shadow}} = \frac{\text{student height}}{\text{length of shadow}}$$

$$\frac{50 \text{ m}}{16 \text{ m}} = \frac{1.5 \text{ m}}{x}$$

$$x \times \frac{50 \text{ m}}{16 \text{ m}} = 1.5 \text{ m}$$

$$x = 1.5 \text{ m} \times \frac{16 \text{ m}}{50 \text{ m}}$$

$$x = 0.48 \text{ m or } 48 \text{ cm}$$

25.a. The frog is eating for (24 – 8) hours in a day.

$$\frac{6 \text{ insects}}{1 \text{ hour}} = \frac{f}{16 \text{ hours}}$$

$$f = 96$$

The dragonfly is eating (24 – 13) hours in a day.

$$\frac{9 \text{ insects}}{1 \text{ hour}} = \frac{d}{11 \text{ hours}}$$

$$d = 99 \text{ insects per day}$$

b. Use this ratio: $\frac{99 \text{ insects}}{1 \text{ day}} = \frac{w}{7 \text{ days}}$.

c. Use this ratio: $\frac{96 \text{ insects}}{1 \text{ day}} = \frac{m}{31 \text{ days}}$.

SC 1.

a. Compare your answers with those given on page 488 of your textbook.

b. You should have checked your answers on page 488 of your textbook. Be sure to ask your teacher about a question if the answer does not make sense to you.

SC 2.

a. Compare your answers with those given on page 483 of your textbook.

b. The answers to these questions are provided on pages 483 and 484. Remember to ask your teacher for assistance if a question gives you difficulty.

SC 3.

1. A

2. B

3. C

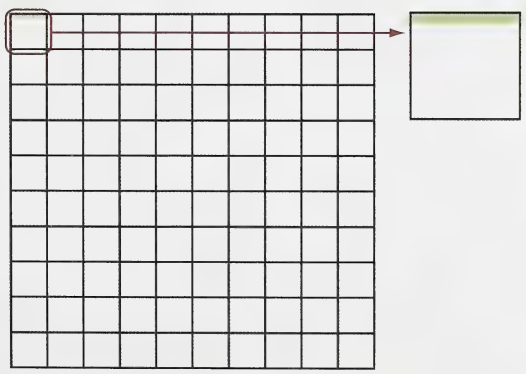
4. D

5. B

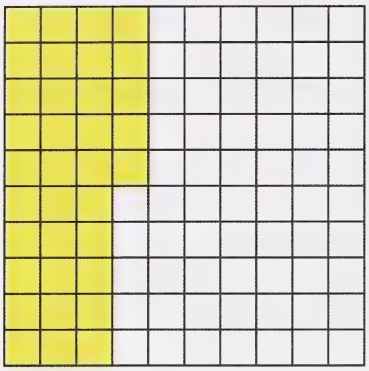
6. 130%

7. 0.4%

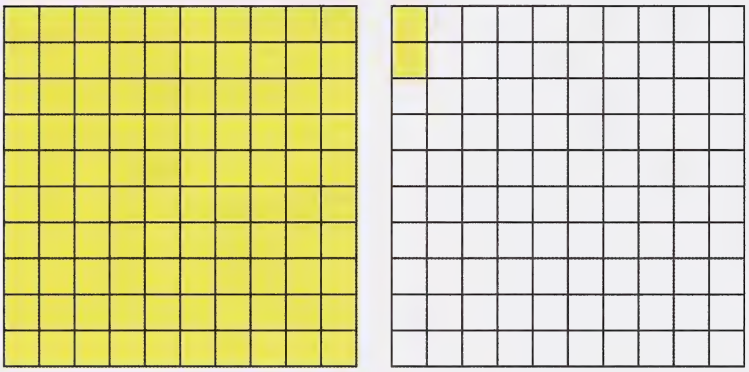
8.a. 0.1%



b. 35%



c. 102%



9.a.

$$15\% = 0.15$$
$$= \frac{15}{100}$$
$$= \frac{3}{20}$$

b.

$$1.24 = 124\%$$
$$= \frac{124}{100}$$
$$= \frac{31}{25}$$

c.

$$\frac{13}{25} = \frac{52}{100}$$
$$= 0.52$$
$$= 52\%$$

10. The agent makes \$24 610, as the following calculations show.

$$\$423\,000 = \$250\,000 + \$173\,000$$
$$5\% \text{ of } \$250\,000 = 0.05 \times \$250\,000$$
$$= \$12\,500$$
$$7\% \text{ of } \$173\,000 = 0.07 \times \$173\,000$$
$$= \$12\,110$$
$$\$12\,500 + \$12\,110 = \$24\,610$$

11. The population was 50 350 after one year and 50 702 after two years.

$$50\,000 \times 100.7\% = 50\,350$$
$$50\,350 \times 100.7\% = 50\,702$$

12. Helen paid \$7.80 in taxes.

$$\$64.98 \times (5\% + 7\%) = \$64.98 \times 12\%$$
$$= \$64.98 \times 0.12$$
$$= \$7.80$$

b. Helen paid a total price of \$64.98+\$7.80=\$72.78.

13.a. The club earns \$1500 on sales of \$6000.

$$\$6000 \times 25\% = \$1500$$

b. The parent committee will donate \$1875.

$$\$1500 \times 125\% = \$1500 \times 1.25$$
$$= \$1875$$

c. The club will receive a total of \$3375.

14.a.

Year	0	1	2	3	4	5	6
% of ice cover	70%	65.80%	61.85%	58.14%	54.65%	51.37%	48.29%
formula		b1*0.94	c1*0.94	d1*0.94	e1*0.94	f1*0.94	g1*0.94

There will be 65.8% ice cover after one year.

b. After the third year, there will be 58.1% ice cover.

c. It will take six years before there is less than 50% ice cover.

SC 4.

1. B

2. C

3. D

4. B

5. For $300 in Canadian money, you would receive $360 in Australian money.

$$\frac{\$500C}{\$600A} = \frac{\$300C}{x}$$

$$x = \$300C \times \frac{\$600A}{\$500C}$$

$$x = \$360C$$

6. The bakery should make 8000 loaves of white bread and 4000 loaves of rye bread.

$$\frac{\text{brown}}{\text{white}} = \frac{5}{2} \qquad \frac{\text{brown}}{\text{rye}} = \frac{5}{1}$$

$$\frac{20\,000}{\text{white}} = \frac{5}{2} \qquad \frac{20\,000}{\text{rye}} = \frac{5}{1}$$

$$\text{white} = 20\,000 \times \frac{2}{5} \qquad \text{rye} = 20\,000 \times \frac{1}{5}$$

$$= 8\,000 \qquad = 4\,000$$

7. She can make 63 scarves from 28 m of fabric.

$$\frac{\text{scarves}}{\text{fabric}} = \frac{9}{4}$$
$$\frac{\text{scarves}}{28} = \frac{9}{4}$$
$$\text{scarves} = \frac{9}{4} \times 28$$
$$= 63$$

8. There are 25 litres of maples syrup in tank B.

$$\frac{A}{B} = \frac{2}{5}$$
$$\frac{10L}{B} = \frac{2}{5}$$
$$B = \frac{5}{2} \times 10L$$
$$= 25L$$

9.a. One-eighth choose other; that is, four students.

b. The ratio of cat : rabbit is 16 : 4. One equivalent ratio is 4 : 1.

10. The area of the top of the box is 1024 cm^2.

$$\frac{B}{C} = \frac{2}{3} \qquad \frac{A}{C} = \frac{8}{3}$$
$$\frac{B}{24\text{ cm}} = \frac{2}{3} \qquad \frac{A}{24\text{ cm}} = \frac{2}{3}$$
$$B = \frac{2}{3} \times 24\text{ cm} \qquad A = \frac{8}{3} \times 24\text{ cm}$$
$$= 16\text{ cm} \qquad = 64\text{ cm}$$

$$\text{Area} = L \times W$$
$$= 16\text{ cm} \times 64\text{ cm}$$
$$= 1024\text{ cm}^2$$

11.a. The 500-mL bottle is a better buy.

$$\frac{210}{500} = \frac{21}{50}$$
$$\frac{330}{750} = \frac{22}{50}$$

b. The 500-mL bottle is the better buy.

$$\frac{\$2.09}{500 \text{ mL}} = \$0.00418 / \text{mL}$$
$$\frac{\$3.29}{750 \text{ mL}} = \$0.0043867 / \text{ mL}$$

12.a. Peter's sister ran faster since she ran over 8 m per second while Peter ran less than 7 metres per second.

b. In two minutes, Peter would run 800 m (four times as far as in 30 seconds). In two minutes, Peter's sister would run 1000 m (three and one-third times as far as in 36 seconds).

c. It would take Peter's sister two minutes to run 1000 m. It would take Peter two and one-half minutes to run 1000 m. The assumption is that they both keep running at the same rate.

13.a. Karen's rate is \$12/h, and Liam's rate is \$11/h.

$$\frac{\$420}{35 \text{ h}} = \$12 / \text{h}$$
$$\frac{\$440}{40 \text{ h}} = \$11 / \text{ h}$$

b. Liam earns \$88 in an eight-hour shift.

14.a. The measured height in the picture is 5 cm. The ratio of the measured height to the actual height is 5 : 114.

b. The horn in the picture measures 8 mm. The actual length is 18.24 cm.

$$\frac{8 \text{ mm}}{x} = \frac{5}{114}$$
$$c = \frac{114}{5} \times 8 \text{ mm}$$
$$c = 182.4 \text{ mm or } 18.24 \text{ cm}$$

© 2009